$$\lim_{\substack{x \to -\infty \\ \to +\infty}} \frac{e^{x}\left(1+e^{-x}\right)^{x}}{2\sin(e^{x}) - 2\cos\left(e^{\frac{x}{2}}\right) + 2 - 3e^{x}} =$$

$e^{x} = y$, $x \longrightarrow -\infty$

$$\lim_{y \to 0^{+}} \frac{y\left(1+\frac{1}{y}\right)^{\log(y)}}{2\sin(y) - 2\cos(\sqrt{y}) + 2 - 3y}$$

$N(y)$

$D(y)$

$$D(y) = 2\left(y + o(y^{2})\right) - 2\left(1 - \frac{1}{2}(\sqrt{y})^{2} + o\left(\sqrt{y}^{3}\right)\right)$$
$$+ 2 - 3y$$

$$= 2y + o(y^{2}) - 2 + y + o(y^{3/2}) + 2 - 3y$$

$$= o(y^{3/2}), \quad y \to 0^{+}$$

$$D(y) = 2y + o(y^2) - 2\left(1 - \frac{1}{2}y + \frac{1}{4!}(\sqrt{y})^4 + o(y^{5/2})\right)$$

$$+2 - 3y$$

$$= 2y + o(y^2) - 2 + y - \frac{1}{12}y^2 + o(y^{5/2})$$

$$+2 - 3y = -\frac{1}{12}y^2 + o(y^2), \quad y \to 0^+$$

$$N(y) = y\left(1 + \frac{1}{y}\right)^{\log(y)} =$$

$$= y\,\frac{1}{(y)^{\log(y)}}\,(1+y)^{\log(y)}$$

$$= y^{1-\log(y)}\left((1+y)^{\frac{1}{y}}\right)^{y\log(y)}$$

$$=$$

$$= y^{1-\log(y)}\left(e+o(1)\right)^{o(1)} =$$

$$= y^{1-\log(y)}\left(1+o(1)\right)$$

$$\lim_{y\to 0^+}\frac{N(y)}{D(y)} = \lim_{y\to 0^+}\frac{y^{1-\log(y)}\left(1+o(1)\right)}{-\frac{1}{12}y^2\left(1+o(1)\right)} =$$

$$= -12\lim_{y\to 0^+} y^{-1-\log(y)}$$

$$= -12\left(0^+\right)^{+\infty} = 0^-$$

NOTA: $(0^+)^{+\infty} = 0^+$ Infatti

$$f(x)\xrightarrow[x\to x_0]{} 0^+,\quad \log(f(x))\xrightarrow[x\to x_0]{} -\infty$$

Se $\displaystyle g(x) \xrightarrow[x \to x_0]{} +\infty$, allora

$$\left(f(x)\right)^{g(x)} = e^{g(x)\log(f(x))} \xrightarrow[x \to x_0]{} e^{-\infty} = 0^+$$

$$x \longleftrightarrow x$$

$$e^x = y, \quad x \to +\infty$$

$$\lim_{y \to +\infty} \frac{y\left(1+\frac{1}{y}\right)^{\log(y)}}{2\sin(y) - 2\cos(\sqrt{y}) - 2 + 3y} =$$

$$\lim_{y \to +\infty} \frac{\cancel{y}\left(\left(1+\frac{1}{y}\right)^{y}\right)^{\frac{\log(y)}{y}}}{3\cancel{y}\left(1+o(1)\right)} = \frac{e^0}{3} = \frac{1}{3}$$

$$f(x) = \arccos\left(\log^2(x) - |\log(x)| \right)$$

$$\begin{cases} -1 \leq \log^2(x) - \log(x) \leq 1 \\ \log(x) \geq 0 \end{cases} \quad (\mathrm{I})$$

$$\begin{cases} -1 \leq \log^2(x) + \log(x) \leq 1 \\ \log(x) < 0 \end{cases} \quad (\mathrm{II})$$

$$\begin{cases} y^2 - y - 1 \leq 0 \\ y^2 - y + 1 \geq 0 \\ y \geq 0 \end{cases} \quad (\mathrm{I})$$

$$y = \log(x)$$

$$y^2 - y + 1 > 0 \quad \forall y$$

$$(\Delta = 1 - 4 < 0)$$

NOTA: $y^2 - y + 1 \geq 0$, $y = \log(x) \geq 0$

equivale a $\log^2(x) - \log(x) \geq -1$

QUINDI se $\log(x) \geq 0$ l'argomento

dell'arccos è SEMPRE MAGGIORE

di -1

$y^2 - y - 1 \leq 0$, $y_- \leq y \leq y_+$

$$y_\pm = \frac{1 \pm \sqrt{5}}{2}$$

$$y_- = \frac{1 - \sqrt{5}}{2} < -\frac{1}{2}$$

$$0 \leq y \leq \frac{1 + \sqrt{5}}{2}$$

NOTA: IN PARTICOLARE

$$y^2 - y = 1 \iff y = \frac{1+\sqrt{5}}{2}$$

$$0 \leq \log(x) \leq \frac{1+\sqrt{5}}{2}$$

$$1 \leq x \leq e^{\frac{1+\sqrt{5}}{2}}$$

e $\log^2(x) - \log(x) = 1 \iff x = e^{\frac{1+\sqrt{5}}{2}}$

$$\begin{cases} -1 \leq \log^2(x) - \log(x) \leq 1 \\ \log(x) \geq 0 \end{cases} \quad (\mathrm{I})$$

$$\iff 1 \leq x \leq e^{\frac{1+\sqrt{5}}{2}}$$

e

$$\log^2(x) - \log(x) = 1 \iff x = e^{\frac{1+\sqrt{5}}{2}}$$

$$\log^2(x) - \log(x) > -1 \quad \forall x \in \left[1, e^{\frac{1+\sqrt{5}}{2}}\right]$$

Ex

$$\begin{cases} -1 \leq \log^2(x) + \log(x) \leq 1 \\ \log(x) < 0 \end{cases} \quad (\mathrm{II})$$

$$e^{-\frac{1+\sqrt{5}}{2}} \leq x < 1 \quad e$$

$$\log^2(x) + \log(x) = 1 \iff x = e^{-\frac{1+\sqrt{5}}{2}}$$

$$\log^2(x) + \log(x) > -1 \quad \forall x \in \left[e^{-\frac{1+\sqrt{5}}{2}}, 1\right)$$

$$\mathrm{dom}(f) = \left[e^{-\frac{1+\sqrt{5}}{2}}, e^{\frac{1+\sqrt{5}}{2}}\right]$$

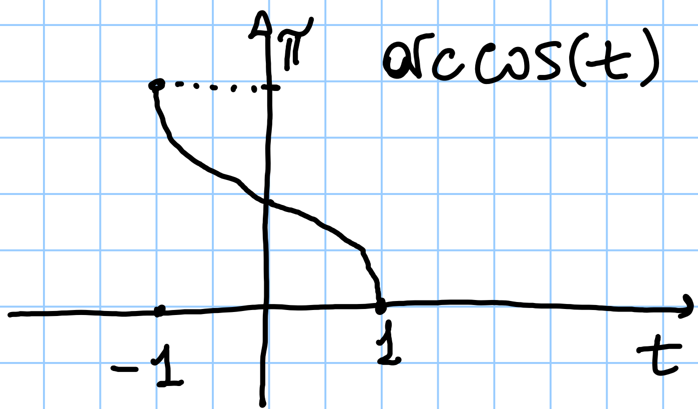
$$f\left(e^{-\frac{1+\sqrt{5}}{2}}\right) = \arccos(1) = 0$$

$$f\left(e^{\frac{1+\sqrt{5}}{2}}\right) = \arccos(1) = 0$$

$$e \quad f(x) = 0 \iff x = e^{\pm\frac{1+\sqrt{5}}{2}}$$

$$e \quad f(x) < \arccos(-1) = \pi$$

RICORDARE



$$f(x)=\begin{cases}\arccos\left(\log^2(x)-\log(x)\right), & x\in\left[1,e^{\frac{1+\sqrt5}{2}}\right]\\ \arccos\left(\log^2(x)+\log(x)\right), & x\in\left[e^{-\frac{1+\sqrt5}{2}},1\right)\end{cases}$$

$$f'(x)=\begin{cases}-\dfrac{1}{\sqrt{1-\left(\log^2(x)-\log(x)\right)^2}}\cdot\left(2\log(x)\cdot\dfrac{1}{x}-\dfrac{1}{x}\right) & x\in\left(1,e^{\frac{1+\sqrt5}{2}}\right)\\ \dfrac{-1}{\sqrt{1-\left(\log^2(x)+\log(x)\right)^2}}\cdot\left(2\log(x)\dfrac{1}{x}+\dfrac{1}{x}\right) & x\in\left(e^{-\frac{1+\sqrt5}{2}},1\right)\end{cases}$$

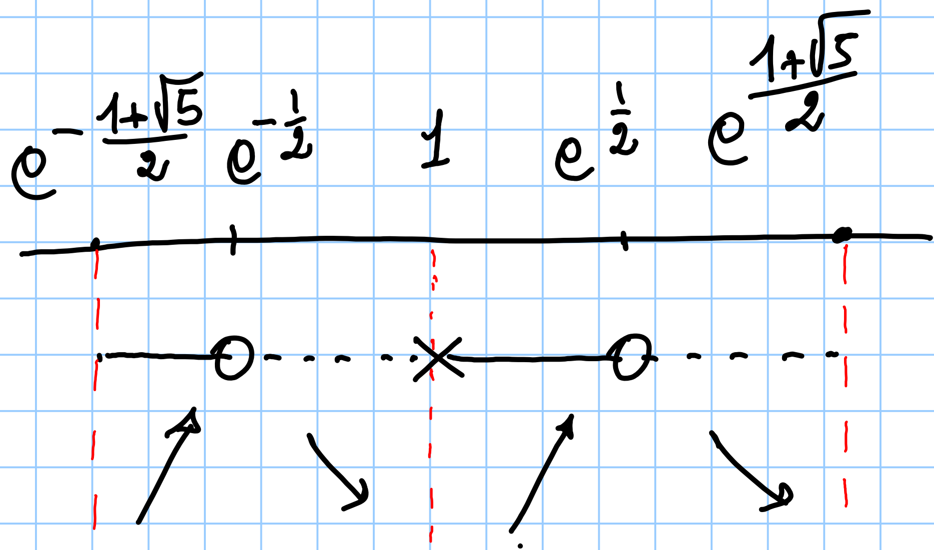
$$f'(x)\geq 0 \iff 2\log(x)\cdot\frac{1}{x}-\frac{1}{x}\leq 0$$
$$x\in\left(1,e^{\frac{1+\sqrt5}{2}}\right)$$

$\Longleftrightarrow \quad 2\log(x) - 1 \leq 0 \quad \Longleftrightarrow$

$x \leq e^{\frac{1}{2}} = \sqrt{e}$

$f'(x) \geq 0 \quad \Longleftrightarrow \quad x \leq \frac{1}{\sqrt{e}}$

$x \in \left(e^{-\frac{1+\sqrt{5}}{2}}, 1\right)$



$x = \frac{1}{\sqrt{e}}$ max locale

$$f\left(\frac{1}{\sqrt{e}}\right) = \arccos\left(\frac{1}{4} - \frac{1}{2}\right) = \arccos\left(-\frac{1}{4}\right)$$

$x = \sqrt{e}$ max locale

$$f(\sqrt{e}) = \arccos\left(\frac{1}{4} - \frac{1}{2}\right) = \arccos\left(-\frac{1}{4}\right)$$

$x = 1$ min locale

$$f(1) = \arccos(0) = \frac{\pi}{2}$$

$$\lim_{x \to 1^+} f'(x) = 1 \quad , \quad \lim_{x \to 1^-} f'(x) = -1$$

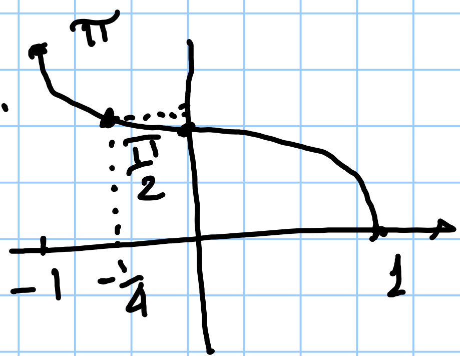
$X = 1$ PUNTO ANGOLOSO

$$\lim_{x \to \left(e^{\frac{1+\sqrt{5}}{2}}\right)^-} f'(x) = -\frac{1}{0^+} e^{-\frac{1+\sqrt{5}}{2}}(\sqrt{5}) = -\infty$$

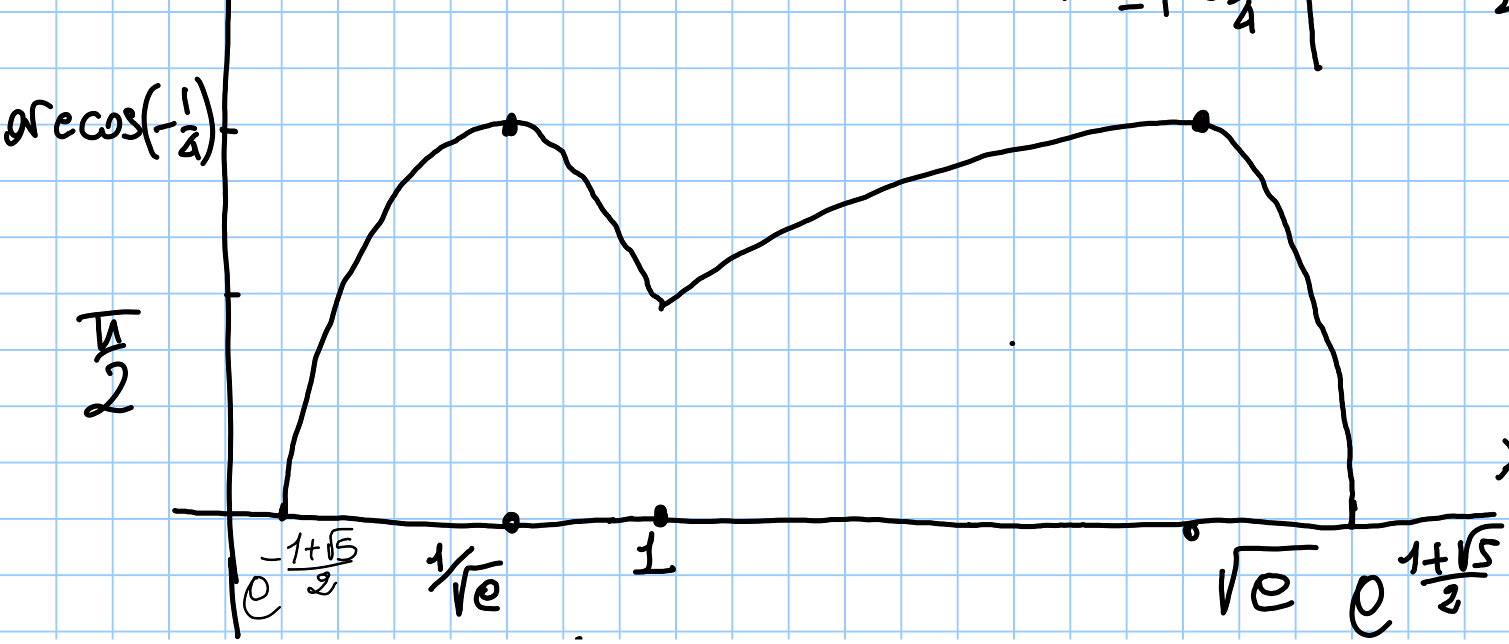
Ex

$$\lim_{x \to \left(e^{-\frac{1+\sqrt{5}}{2}}\right)^+} f'(x) = -\frac{1}{0^+} e^{\frac{1+\sqrt{5}}{2}}(-\sqrt{5}) = +\infty$$

$X = e^{-\frac{1+\sqrt{5}}{2}}$ e $x = e^{\frac{1+\sqrt{5}}{2}}$ PUNTI di TANGENZA VERTICALE.



$\arccos(\log^2(x) - |\log(x)|)$



$$\lim_{\substack{x \to +\infty \\ x \to 0^+}} \frac{1 - \sqrt[3]{\cos(x \sin(x))} - 2x^4}{x^3 \log(x - \mathrm{Tgh}(x)) + 3x^2}$$

$$f(x) \underset{x \to +\infty}{=} \frac{-2x^4(1 + o(1))}{x^3 \log(x - 1 + o(1)) + 3x^2} =$$

$$= \frac{-2x^4(1 + o(1))}{x^3 \log(X - 1 + o(1))(1 + o(1))} \underset{x \to +\infty}{\longrightarrow} -\infty$$

$$f(x) = \frac{N(x)}{D(x)}$$

$$Tgh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh(x)}{\cosh(x)} =$$

$$= \frac{x + o(x^2)}{1 + \frac{x^2}{2} + o(x^3)} = (x + o(x^2))\left(1 + \frac{x^2}{2} + o(x^3)\right)^{-1}$$

$$= (x + o(x^2))\left(1 - \frac{x^2}{2} + o(x^3) + o(x^2)\right) =$$

$$= (x + o(x^2))\left(1 - \frac{x^2}{2} + o(x^2)\right) =$$

$$= x + o(x^2)$$

$$D(x) = x^3 \log(x - Tgh(x)) + 3x^2$$

$$= x^3 \log\left(x - x + o(x^2)\right) + 3x^2 =$$

$$= x^3 \log(\overset{?}{o}(x^2)) + 3x^2$$

$$\mathrm{Tgh}(x) = \frac{x + \frac{x^3}{3!} + o(x^4)}{1 + \frac{x^2}{2!} + o(x^3)} =$$

$$= \left(x + \frac{x^3}{3!} + o(x^4)\right)\left(1 + \frac{x^2}{2!} + o(x^3)\right)^{-1}$$

$$= \left(x + \frac{x^3}{3!} + o(x^4)\right)\left(1 - \frac{x^2}{2} + o(x^3)\right) =$$

$$x - \frac{x^3}{2} + \frac{x^3}{6} + o(x^3) = x - \frac{x^3}{3} + o(x^3)$$

$$D(x) = x^3 \log(x - \mathrm{Tgh}(x)) + x^2$$

$$= x^3 \log\left(\frac{x^3}{3} + o(x^3)\right) + 3x^2$$

$$= 3x^2\left(1 + \frac{x}{3}\log\left(\frac{x^3}{3} + o(x^3)\right)\right)$$

$$= 3x^2(1 + o(1))$$

$$N(x) = 1 - (\cos(x\sin(x)))^{\frac{1}{3}} - 2x^4 =$$

$$\cos(x\sin(x)) = \cos(x(x+o(x^2)))$$
$$= \cos(x^2+o(x^3)) =$$
$$1-\frac{1}{2}(x^2+o(x^3))^2+o((x^2+o(x^3))^3)$$
$$= 1-\frac{x^4}{2}+o(x^5)$$
$$(\cos(x\sin(x)))^{\frac{1}{3}} = \left(1-\frac{x^4}{2}+o(x^5)\right)^{\frac{1}{3}} =$$
$$= 1-\frac{1}{6}x^4+o(x^5)$$
$$N(x) = 1-\left(1-\frac{1}{6}x^4+o(x^5)\right)-2x^4$$
$$=\left(\frac{1}{6}-2\right)x^4+o(x^5)$$
$$f(x) = \frac{-\frac{11}{6}x^4(1+o(1))}{3x^2(1+o(1))} = 0^-$$
$$x\to0^+$$

$$f(x) = x^2 e^{\frac{x}{|x|-1}}$$

$$\mathrm{dom}(f) = \mathbb{R} \setminus \{-1, 1\}$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} x^2 e^{\frac{\cancel{x}}{\cancel{x}} \frac{1}{1+o(1)}} = +\infty$$

$$f(x) \underset{x \to +\infty}{=} x^2 e^{1+o(1)}$$

NO AS. OBLIQUO

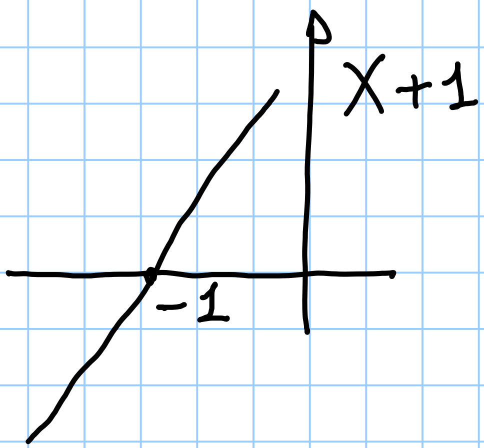
$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^2 e^{\frac{x}{-x} \frac{1}{1+o(1)}} =$$

$$= \lim_{x \to -\infty} x^2 e^{-1+o(1)} = +\infty$$

$$f(x) \underset{x \to -\infty}{=} x^2 e^{-1+o(1)}$$

NO AS. OBLIQUO

$$\lim_{x \to (-1)^-} f(x) = \lim_{x \to (-1)^-} (1+o(1))^2 e^{\frac{-1+o(1)}{-x-1}} =$$



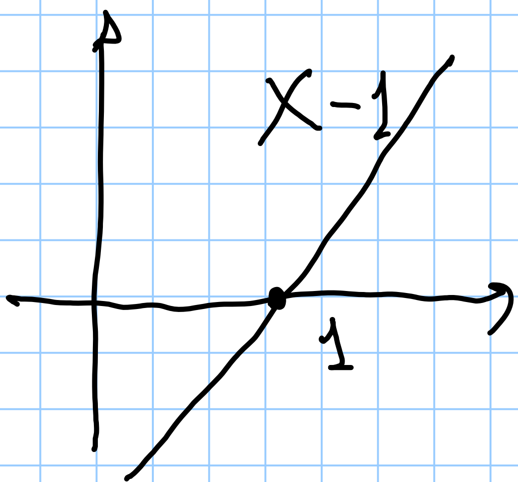
$$= \lim_{x \to (-1)^-} (1+o(1))^2 e^{\frac{1+o(1)}{x+1}}$$

$$= 1 \cdot e^{\frac{1}{0^-}} = e^{-\infty} = 0^+$$

$$\lim_{x \to (-1)^+} f(x) = \lim_{x \to (-1)^+} (1+o(1))^2 e^{\frac{1+o(1)}{x+1}}$$

$$= 1 \cdot e^{1/0^+} = e^{+\infty} = +\infty$$

$$\lim_{x \to (1)^-} f(x) = \lim_{x \to (1)^-} (1+o(1))^2 e^{\frac{x}{x-1}} =$$

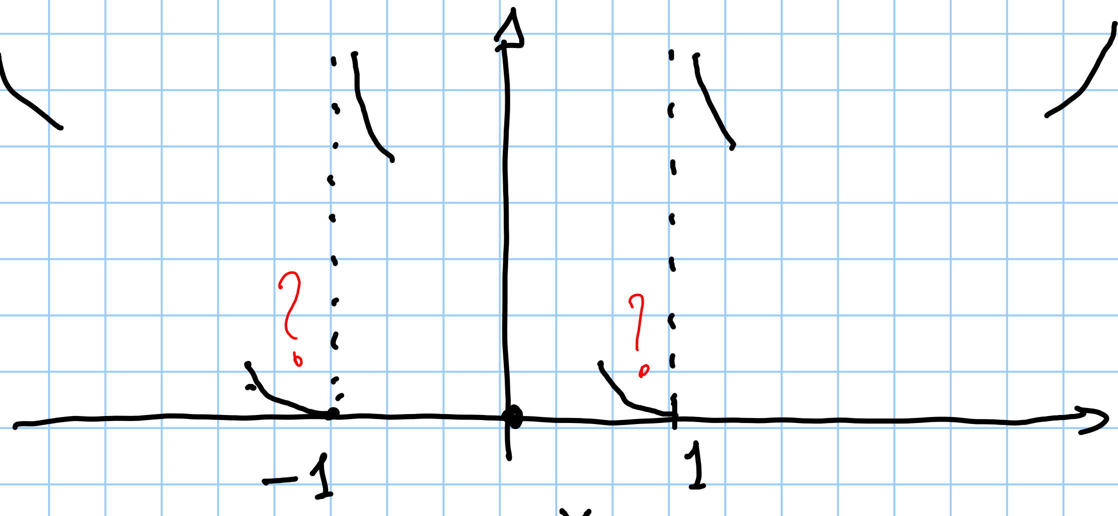


$$= \lim_{x \to (1)^-} (1+o(1))^2 e^{\frac{1+o(1)}{x-1}} =$$

$$= 1 \cdot e^{1/0^-} = e^{-\infty} = 0^+$$

$$\lim_{x \to (1)^+} f(x) = \lim_{x \to (1^+)} (1+o(1))^2 e^{\frac{1+o(1)}{x-1}} =$$

$$= 1 \cdot e^{1/0^+} = e^{+\infty} = +\infty$$



$$f(x) = \begin{cases} x^2 e^{\frac{x}{x-1}}, & x > 0,\ x \neq 1 \\ x^2 e^{-\frac{x}{x+1}}, & x < 0,\ x \neq -1 \end{cases}$$

$$f'(x) = 2x e^{x/(x-1)} + x^2 e^{x/(x-1)} \frac{x-1-x}{(x-1)^2} =$$

$x > 0$
$x \neq 1$

$$= x e^{x/(x-1)} \left(2 - \frac{x}{(x-1)^2} \right)$$

$$= \frac{x\, e^{\frac{x}{x-1}}}{(x-1)^2}\left(2x^2 - 4x + 2 - x\right)$$

$$= \frac{x\, e^{\frac{x}{x-1}}}{(x-1)^2}\left(2x^2 - 5x + 2\right)$$

$$f'(x) \geq 0 \iff 2x^2 - 5x + 2 \geq 0$$

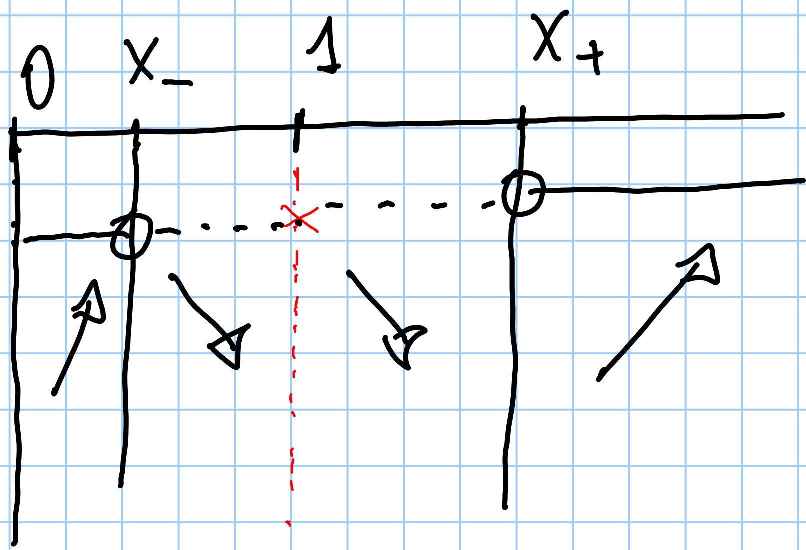
$x > 0,\ x \neq 1$

$$x_\pm = \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$= \frac{5 \pm 3}{4}$$

$$x_+ = 2$$

$$x_- = 1/2$$



$x_- = \frac{1}{2}$ max locale $f\left(\frac{1}{2}\right) = \frac{1}{4}e$

$x_+ = 2$ min locale $f(2) = 4e^2$

$$f'(x) = 2x e^{-\frac{x}{x+1}} + x^2 e^{-\frac{x}{x+1}} (-1) \frac{x+1-x}{(x+1)^2} =$$

$x < 0$
$x \neq -1$

$$= x e^{-\frac{x}{x+1}} \left(2 - \frac{x}{(x+1)^2} \right) =$$

$$= \frac{x e^{-\frac{x}{x+1}}}{(x+1)^2} \left(2x^2 + 2x + 2 - x \right)$$

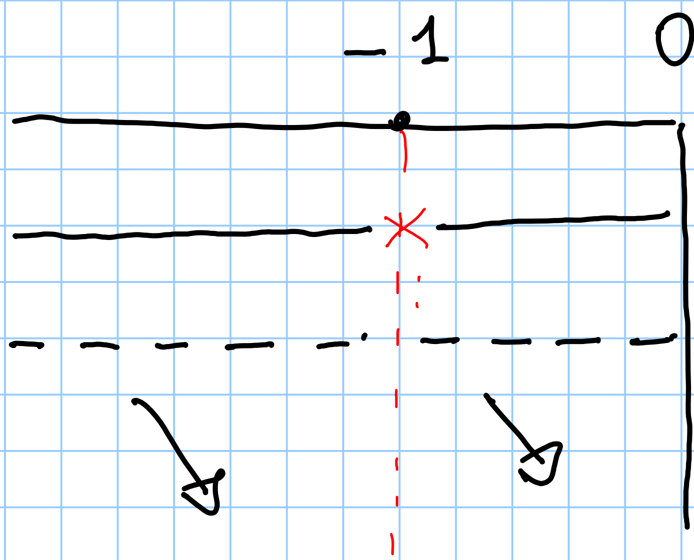
$$= \frac{x e^{-\frac{x}{x+1}}}{(x+1)^2} \left(2x^2 + x + 2 \right)$$

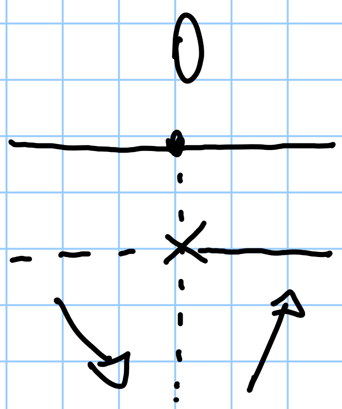
$$2x^2 + x + 2 \geq 0$$

$x < 0,\ x \neq 1$ $\quad$ $\Delta = 1 - 16 < 0$

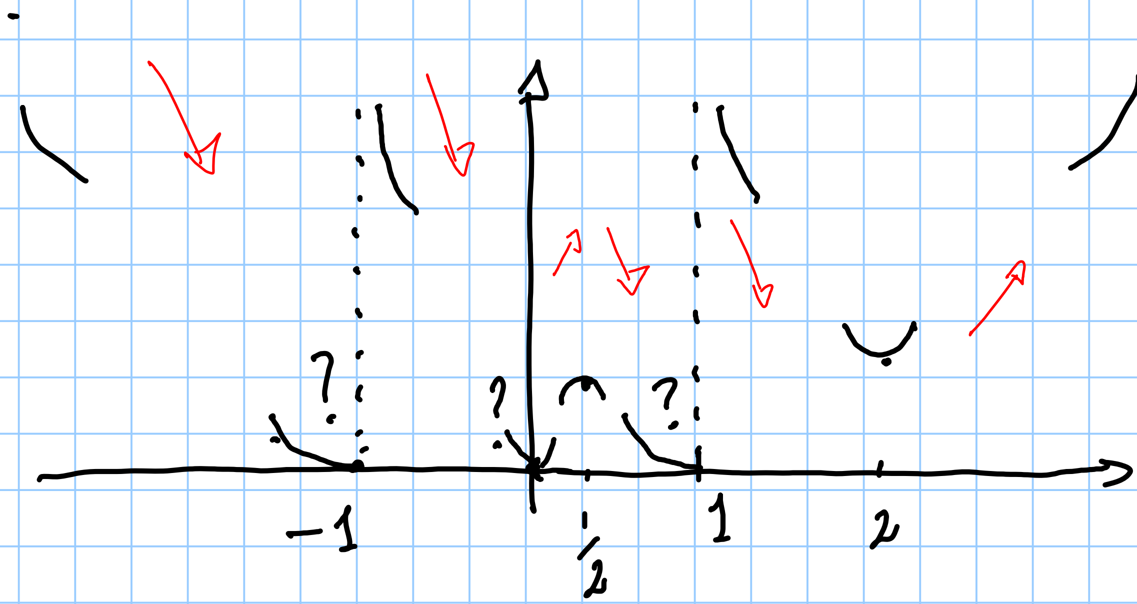
$$2x^2 + x + 2 > 0$$

$$\forall x \in \mathbb{R}$$





$x_0 = 0$ min locale $f(0) = 0$



$$\lim_{x\to(-1)^-} f'(x) =$$

$$\lim_{x\to(-1)^-} \frac{x e^{-\frac{x}{x+1}}}{(x+1)^2}\left(2x^2+x+2\right) =$$

$$\lim_{x\to(-1)^-} \left(-3+o(1)\right) \frac{e^{\frac{1+o(1)}{x+1}}}{(x+1)^2} =$$

$$= -3 \lim_{x \to (-1)^-} \frac{e^{\frac{1}{x+1}(1+o(1))}}{(x+1)^2} =$$

$$y = \frac{1}{x+1} \xrightarrow[x \to (-1)^-]{} \frac{1}{0^-} = -\infty$$

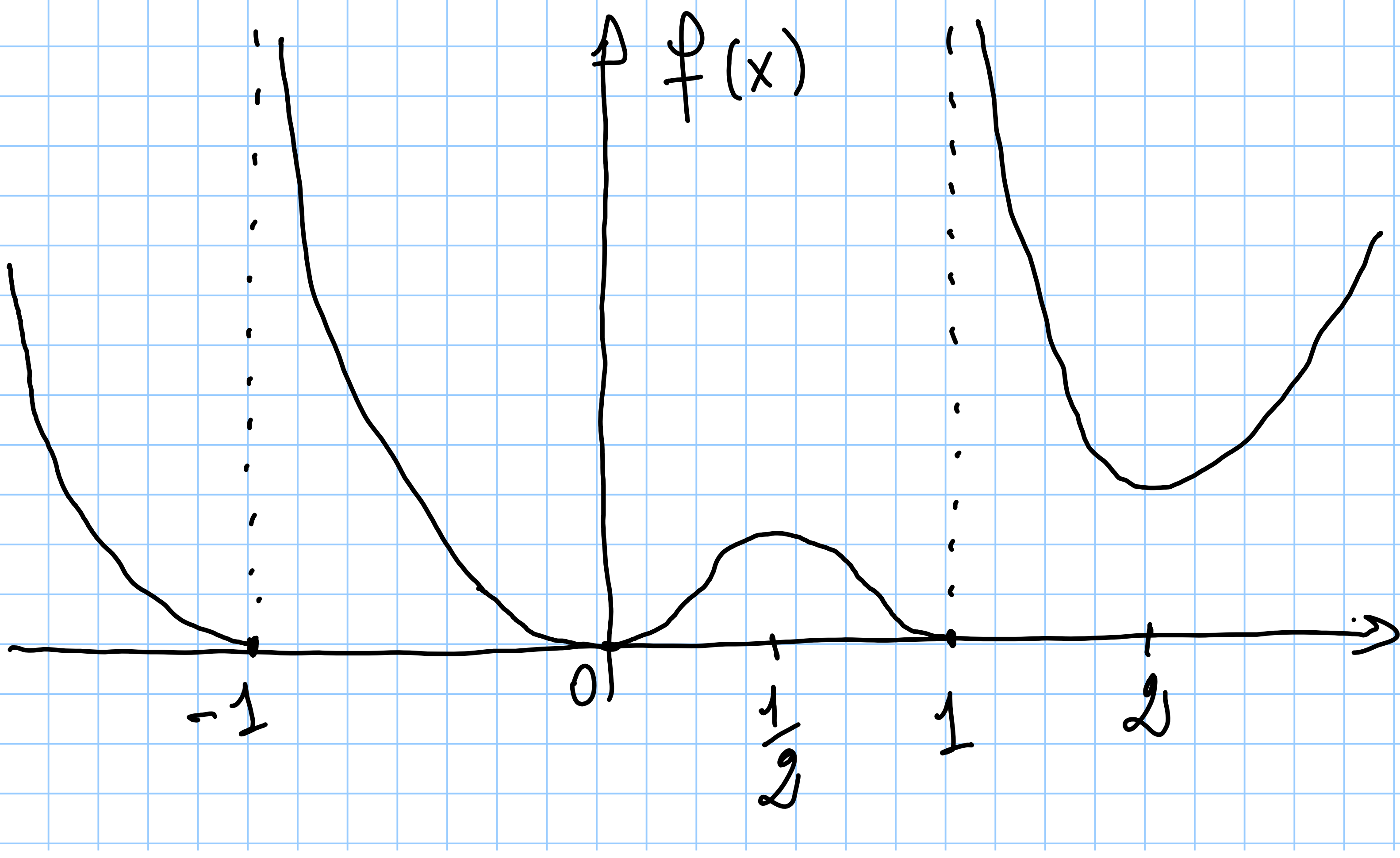
$$= -3 \lim_{y \to -\infty} y^2 e^{y(1+o(1))} =$$

$$= -3 \lim_{t \to +\infty} t^2 e^{-t(1+o(1))} = 0^-$$

Ex $\lim_{x \to 1^-} f'(x) = 0^-$

$$\lim_{x \to 0^+} f'(x) = 0 = \lim_{x \to 0^-} f'(x)$$

T. CONTINUITÀ DERIVATA PRIMA $\Rightarrow f'(0) = 0$



$$\lim_{n} \frac{(1+\sqrt{n})^{\frac{1}{n}} - \frac{\log(n)}{2n} - 1}{\operatorname{arctg}\left(n e^{-n} + \frac{(2n)!}{(n+3)^{2n-1}}\right)}$$

N_n

D_n

FORMULA di STIRLING.

$$(2n)! \underset{n \to +\infty}{=} (2n)^{2n} e^{-2n} \sqrt{4\pi n}\,(1+o(1))$$

$$\frac{(2n)!}{(n+3)^{2n-1}} = \frac{(2n)^{2n} e^{-2n} \sqrt{4\pi n}\,(1+o(1))}{n^{2n-1}\left(1+\frac{3}{n}\right)^{2n-1}}$$

$$= \left(\frac{4}{e^2}\right)^m \frac{m\sqrt{4\pi m}}{\left(1+3/m\right)^{2m}} \cdot \left(1+\frac{3}{m}\right)\left(1+o(1)\right) =$$

$$= \left(4/e^2\right)^m m^{3/2} \cdot \frac{\sqrt{4\pi}}{e^6}\left(1+o(1)\right)$$

$$\lim_m \frac{m e^{-m}}{\left(\frac{4}{e^2}\right)^m m^{3/2}} = \lim_m \frac{1}{\left(4/e\right)^m \sqrt{m}} = 0$$

$$D_m = \operatorname{arctg}\left(\left(\frac{4}{e^2}\right)^m m^{3/2} \frac{\sqrt{4\pi}}{e^6}\left(1+o(1)\right)\right)$$

$$= \left(\frac{4}{e^2}\right)^m m^{3/2} \sqrt{4\pi}/e^6 \left(1+o(1)\right)$$

NOTA: $\operatorname{arctg}(a_n + b_n) =$

$a_n = o(b_n)$

$= \operatorname{arctg}(b_n + o(b_n)) =$

$= \operatorname{arctg}(b_n(1+o(1)))$

$$\operatorname{arctg}(x) \underset{x \to 0}{=} x + o(x^2)$$

$$= b_n(1+o(1)) + o(b_n^2)$$

$$= b_n(1+o(1))$$

$$\boxed{D(\operatorname{arctg}(x))\Big|_{x=0} = \frac{1}{1+x^2}\Big|_{x=0} = 1}$$

$$\left(1+\sqrt{n}\right)^{\frac{1}{n}} = e^{\frac{1}{n}\log\left(1+\sqrt{n}\right)}$$

$$\log\left(1+\sqrt{n}\right) = \log\left(\sqrt{n}\left(1+\frac{1}{\sqrt{n}}\right)\right) =$$

$$\log\sqrt{n} + \log\left(1+\frac{1}{\sqrt{n}}\right) =$$

$$\frac{1}{2}\log(n) + \frac{1}{\sqrt{n}} + o\left(\frac{1}{\sqrt{n}}\right)$$

$$\frac{1}{n}\log\left(1+\sqrt{n}\right) = \frac{1}{2}\frac{\log(n)}{n} + \frac{1}{n^{3/2}} + o\left(\frac{1}{n^{3/2}}\right)$$

$$e^{\frac{1}{n}\log(1+\sqrt{n})} = e^{\frac{1}{2}\frac{\log(n)}{n} + \frac{1}{n^{3/2}} + o\left(\frac{1}{n^{3/2}}\right)}$$

$$= 1 + \frac{1}{2}\frac{\log(n)}{n} + \frac{1}{n^{3/2}} + o\left(\frac{1}{n^{3/2}}\right)$$

$$+ \frac{1}{2}\left(\frac{1}{2}\frac{\log(n)}{n} + o\left(\frac{1}{n}\right)\right)^2$$

$$+ o\left(\frac{\log(n)}{n}\right)^2 =$$

$$= 1 + \frac{1}{2}\frac{\log(n)}{n} + \frac{1}{n^{3/2}} + o\left(\frac{1}{n^{3/2}}\right)$$

$$N_n = 1 + \frac{1}{2}\frac{\log(n)}{n} + \frac{1}{n^{3/2}} + o\left(\frac{1}{n^{3/2}}\right)$$

$$- \frac{1}{2}\frac{\log(n)}{n} - 1 = \frac{1}{n^{3/2}}\left(1+o(1)\right)$$

$$\lim_n \frac{\frac{1}{n^{3/2}}\left(1+o(1)\right)}{\left(\frac{4}{e^2}\right)^n n^{3/2}\sqrt{\frac{4\pi}{e^6}}\left(1+o(1)\right)} =$$

$$\frac{e^6}{\sqrt{4\pi}} \lim_n \frac{\left(\frac{e^2}{4}\right)^n}{n^3} = +\infty$$

RICORDARE: $\forall \beta$

$$a>1 \Rightarrow \lim_n \frac{a^n}{n^\beta} = +\infty$$

$$\lim_{n} \frac{\left(\frac{n+1}{n-1}\right)^{\frac{1}{n}} + \overbrace{\frac{n^2+1}{1-n^2}}^{N_n}}{\left|\log\left(\underbrace{\sqrt[4]{n+ne^{-n}} - n^{\frac14}\cos(e^{-n})}_{B}\right)\right|^{\alpha}}$$

$$\boxed{\alpha \in \mathbb{R}}$$

$$\sqrt[4]{n+ne^{-n}} = \sqrt[4]{n}\left(1+e^{-n}\right)^{\frac14} =$$

$$= \sqrt[4]{n}\left(1+\frac14 e^{-n} + o\left(e^{-n}\right)\right)$$

$$\sqrt[4]{n}\cos\left(e^{-n}\right) = \sqrt[4]{n}\left(1 - \frac{e^{-2n}}{2} + o\left(e^{-3n}\right)\right)$$

$$B = n^{\frac14} + \frac{n^{\frac14}}{4} e^{-n} + o\left(n^{\frac14} e^{-n}\right)$$

$$- n^{\frac14} + \frac{n^{\frac14}}{2} e^{-2n} + o\left(n^{\frac14} e^{-3n}\right) \stackrel{\prime}{=}$$

$$= \frac14 \sqrt[4]{n}\, e^{-n}\left(1+o(1)\right)$$

$$|\log(B)|^{\alpha} =$$

$$|\log(\tfrac{1}{4}\sqrt[4]{m}\, e^{-m}(1+o(1)))|^{\alpha} =$$

$$= |\log\tfrac{1}{4} + \tfrac{1}{4}\log(m) - m + \log(1+o(1))|^{\alpha} =$$

$$= m^{\alpha}(1+o(1))^{\alpha} = m^{\alpha}(1+o(1))$$

$$N = \left(\frac{n+1}{n-1}\right)^{\frac{1}{n}} + \frac{n^2+1}{1-n^2}$$

$$\left(\frac{n-1+2}{n-1}\right)^{1/n} = \left(1+\frac{2}{n-1}\right)^{1/n} =$$

$$= e^{\frac{1}{n}\log\left(1+\frac{2}{n-1}\right)} = e^{\frac{1}{n}\left(\frac{2}{n-1}+o\left(\frac{1}{n}\right)\right)}$$

$$1+\frac{1}{n}\cdot\frac{2}{n-1}+o\left(\frac{1}{n^2}\right) = 1+\frac{2}{n^2}\frac{1}{1-\frac{1}{n}}+o\left(\frac{1}{n^2}\right)$$

$$= 1+\frac{2}{n^2}\left(1+\frac{1}{n}\right)+o\left(\frac{1}{n^2}\right) =$$

$$= 1+\frac{2}{n^2}+o\left(\frac{1}{n^2}\right)$$

$$\frac{n^2+1}{1-n^2} = -\frac{1+\frac{1}{n^2}}{1-\frac{1}{n^2}} = -\left(1+\frac{1}{n^2}\right)\left(1-\frac{1}{n^2}\right)^{-1} =$$

$$= -\left(1+\frac{1}{n^2}\right)\left(1+\frac{1}{n^2}+o\left(\frac{1}{n^2}\right)\right)$$

$$= -1-\frac{2}{n^2}+o\left(\frac{1}{n^2}\right)$$

$$N = \cancel{1} + \cancel{\frac{2}{n^2}} + o\left(\frac{1}{n^2}\right) - \cancel{1} - \cancel{\frac{2}{n^2}} = o\left(\frac{1}{n^2}\right) ?$$

$$e^{\frac{1}{n}\log\left(1+\frac{2}{n-1}\right)} = e^{\frac{1}{n}\left(\frac{2}{n-1} - \frac{1}{2}\frac{4}{(n-1)^2} + o\left(\frac{1}{n^2}\right)\right)}$$

$$\begin{cases} \dfrac{2}{n-1} = \dfrac{1}{n}\dfrac{2}{1-\frac{1}{n}} = \dfrac{2}{n}\left(1 + \dfrac{1}{n} + o\left(\dfrac{1}{n}\right)\right) \\ \dfrac{2}{(n-1)^2} = \dfrac{2}{n^2\left(1-\frac{1}{n}\right)^2} = \dfrac{2}{n^2}\left(1 + o(1)\right) \end{cases}$$

$$= e^{\frac{2}{n^2} + \cancel{\frac{2}{n^3}} + o\left(\frac{1}{n^3}\right) - \cancel{\frac{2}{n^3}}} = 1 + \frac{2}{n^2} + o\left(\frac{1}{n^3}\right)$$

$$+ \frac{1}{2}\left(\frac{2}{n^2} + o\left(\frac{1}{n^3}\right)\right)^2 + o\left(\frac{1}{n^4}\right) = 1 + \frac{2}{n^2} + o\left(\frac{1}{n^3}\right)$$

$$\frac{n^2+1}{1-n^2} = -\frac{1+\frac{1}{n^2}}{1-\frac{1}{n^2}} = -\left(1+\frac{1}{n^2}\right)\left(1-\frac{1}{n^2}\right)^{-1} =$$

$$-\left(1+\frac{1}{n^2}\right)\left(1+\frac{1}{n^2}+o\left(\frac{1}{n^3}\right)\right)=$$

$$=-1-\frac{2}{n^2}+o\left(\frac{1}{n^3}\right)$$

$$N=1+\frac{2}{n^2}+o\left(\frac{1}{n^3}\right)-1-\frac{2}{n^2}=$$

$$=o\left(\frac{1}{n^3}\right) \quad ?$$

$$\frac{1}{n}\log\left(1+\frac{2}{n-1}\right)=\frac{1}{n}\left(\frac{2}{n-1}-\frac{1}{2}\frac{4}{(n-1)^2}+\frac{1}{3}\frac{8}{(n-1)^3}+\right.$$

$$\left.+o\left(\frac{1}{n^3}\right)\right)=\frac{1}{n}\left(\frac{2}{n}\left(1-\frac{1}{n}\right)^{-1}-\frac{2}{n^2}\left(1-\frac{1}{n}\right)^{-2}\right.$$

$$\left.+\frac{8}{3}\frac{1}{n^3}\left(1-\frac{1}{n}\right)^{-3}+o\left(\frac{1}{n^3}\right)\right)=$$

$$= \frac{1}{n}\left(\frac{2}{n}\left(1+\frac{1}{n}+\frac{1}{n^2}+o\left(\frac{1}{n^2}\right)\right)\right.$$

$$\left.-\frac{2}{n^2}\left(1+\frac{2}{n}+o\left(\frac{1}{n}\right)\right)+\frac{8}{3}\frac{1}{n^3}\left(1+o(1)\right)\right.$$

$$\left.+o\left(\frac{1}{n^3}\right)\right)=\frac{1}{n}\left(\frac{2}{n}+\cancel{\frac{2}{n^2}}+\frac{2}{n^3}-\cancel{\frac{2}{n^2}}-\frac{4}{n^3}+\frac{8}{3}\frac{1}{n^3}\right.$$

$$\left.+o\left(\frac{1}{n^3}\right)\right)=\frac{2}{n^2}+\left(\frac{8}{3}-2\right)\frac{1}{n^4}+o\left(\frac{1}{n^4}\right)=$$

$$=\frac{2}{n^2}+\frac{2}{3}\frac{1}{n^4}+o\left(\frac{1}{n^4}\right)$$

$$e^{\frac{1}{n}\log\left(1+\frac{2}{n-1}\right)} = e^{2/n^2+\frac{2}{3}\frac{1}{n^4}+o\left(\frac{1}{n^4}\right)} =$$

$$= 1 + \frac{2}{m^2} + \frac{2}{3}\frac{1}{m^4} + o\left(\frac{1}{m^4}\right) +$$

$$\frac{1}{2}\left(\frac{2}{m^2} + \frac{2}{3}\frac{1}{m^4} + o\left(\frac{1}{m^4}\right)\right)^2 + o\left(\frac{1}{m^5}\right) =$$

$$= 1 + \frac{2}{m^2} + \frac{2}{3}\frac{1}{m^4} + \frac{2}{m^4} + o\left(\frac{1}{m^4}\right)$$

$$\frac{m^2+1}{1-m^2} = -\frac{1+\frac{1}{m^2}}{1-\frac{1}{m^2}} = -\left(1+\frac{1}{m^2}\right)\left(1-\frac{1}{m^2}\right)^{-1} =$$

$$-\left(1 + \frac{1}{m^2}\right)\left(1 + \frac{1}{m^2} + \frac{1}{m^4} + o\left(\frac{1}{m^5}\right)\right) =$$

$$= -1 - \frac{2}{m^2} - \frac{1}{m^4} - \frac{1}{m^4} + o\left(\frac{1}{m^5}\right) =$$

$$= -1 - \frac{2}{n^2} - \frac{2}{n^4} + o\left(\frac{1}{n^5}\right)$$

$$N = 1 + \frac{2}{n^2} + \frac{8}{3}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)$$

$$-1 - \frac{2}{n^2} - \frac{2}{n^4} = \frac{2}{3}\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)$$

$$\lim_{n \to +\infty} \frac{N}{D^\alpha} = \lim_{n \to +\infty} \frac{\frac{2}{3}\,{}^{1}/_{n^4}\left(1 + o(1)\right)}{n^\alpha}$$

$$= \begin{cases} \alpha < -4, & +\infty \\ \alpha = -4, & \frac{2}{3} \\ \alpha > -4, & 0^+ \end{cases}$$