$$f(x) = \left(x^2+1-|3x-1|\right)^{\frac{1}{3}} =$$

$$= \begin{cases} \left(x^2-3x+2\right)^{\frac{1}{3}}, & x \geq \frac{1}{3} \\ \left(x^2+3x\right)^{1/3}, & x < \frac{1}{3} \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{3} \dfrac{2x-3}{\left(x^2-3x+2\right)^{\frac{2}{3}}}, & x > \frac{1}{3} \\ & x \neq 1, 2 \\ \frac{1}{3} \dfrac{2x+3}{\left(x^2+3x\right)^{2/3}}, & x < \frac{1}{3} \\ & x \neq -3, 0 \end{cases}$$

$$f'(x) \quad \frac{1}{3} \left(x^2-3x+2\right)^{-\frac{2}{3}} (2x-3), \quad x > \frac{1}{3}$$

$$x \neq 1, 2$$

$$f''(x) = \frac{1}{3}\left(-\frac{2}{3}\right)\left(x^2-3x+2\right)^{-\frac{5}{3}}(2x-3)^2 + \frac{1}{3}\left(x^2-3x+2\right)^{-2/3}\cdot 2 =$$

$$= \frac{1}{3}\,\frac{1}{\left(x^2-3x+2\right)^{5/3}}\left[-\frac{2}{3}(2x-3)^2 + 2\left(x^2-3x+2\right)\right] =$$

$$= \frac{1}{9}\,\frac{1}{\left(x^2-3x+2\right)^{5/3}}\cdot\left[-2\left(4x^2-12x+9\right)+6x^2-18x+12\right] =$$

$$= \frac{1}{9}\,\frac{1}{\left(x^2-3x+2\right)^{5/3}}\left[-2x^2+6x-6\right] =$$

$$= \frac{2}{9}\,\frac{-x^2+3x-3}{\left(x^2-3x+2\right)^{5/3}}$$

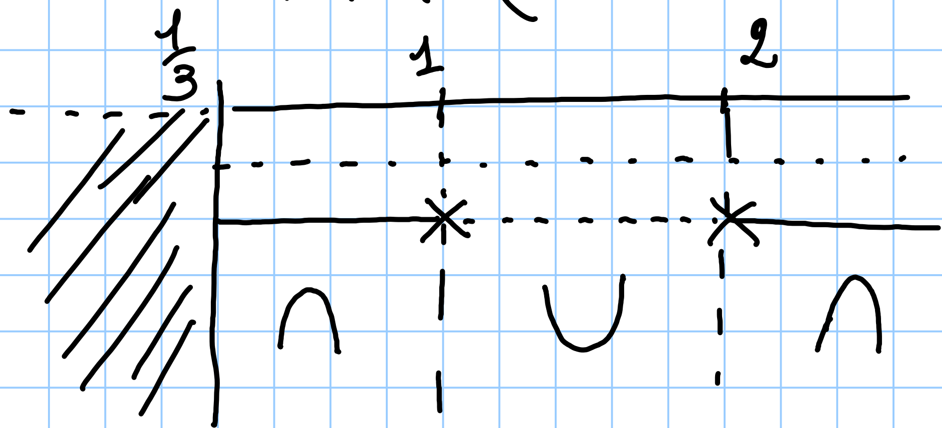
$$f''(x) = \begin{cases} \dfrac{2}{9}\,\dfrac{-x^2+3x-3}{(x^2-3x+2)^{5/3}} & x > \frac{1}{3} \\ & x \neq 1,2 \\ \dfrac{2}{9}\,\dfrac{-x^2-3x-9}{(x^2+3x)^{5/3}} & x < \frac{1}{3} \\ & x \neq 0,3 \end{cases}$$

Ex

$\boxed{x > \frac{1}{3}}$

$-x^2+3x-3 \geq 0$

MAI $(\Delta = 9 - 12 < 0)$



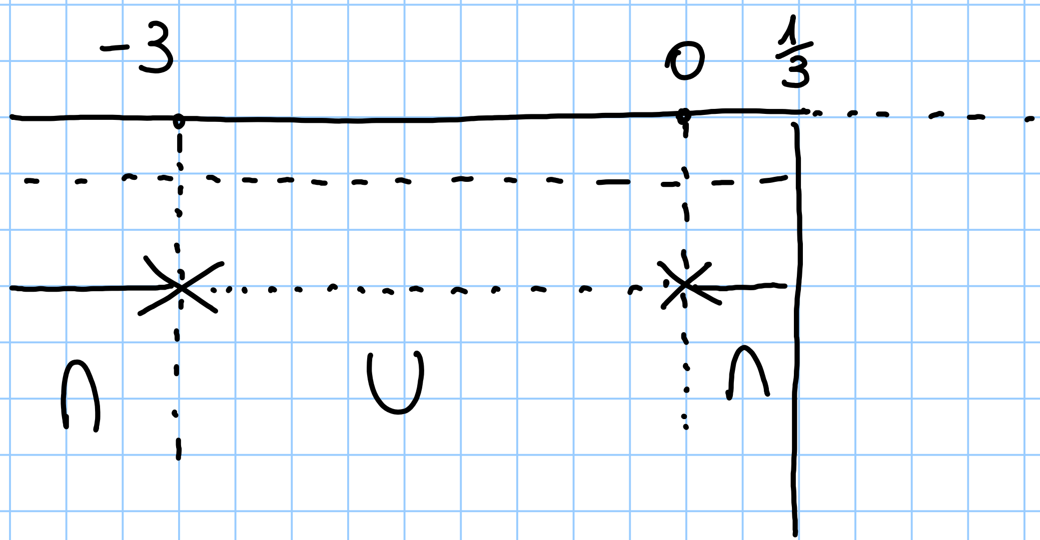
$x^2-3x+2 = (x-1)(x-2)$

f concava in $\left(\frac{1}{3}, 1\right) \cup (2, +\infty)$

convessa in $(1, 2)$

$$\lim_{x \to 1} f'(x) = -\infty \quad ; \quad x = 1 \text{ Flesso (DISCENDENTE)}$$

$$\lim_{x \to 2} f'(x) = +\infty \quad ; \quad x = 2 \text{ Flesso (ASCENDENTE)}$$



$$\boxed{x < \frac{1}{3}}$$

$$-x^2 - 3x - 9 \geq 0$$

MAI $(\Delta = 9 - 36 < 0)$

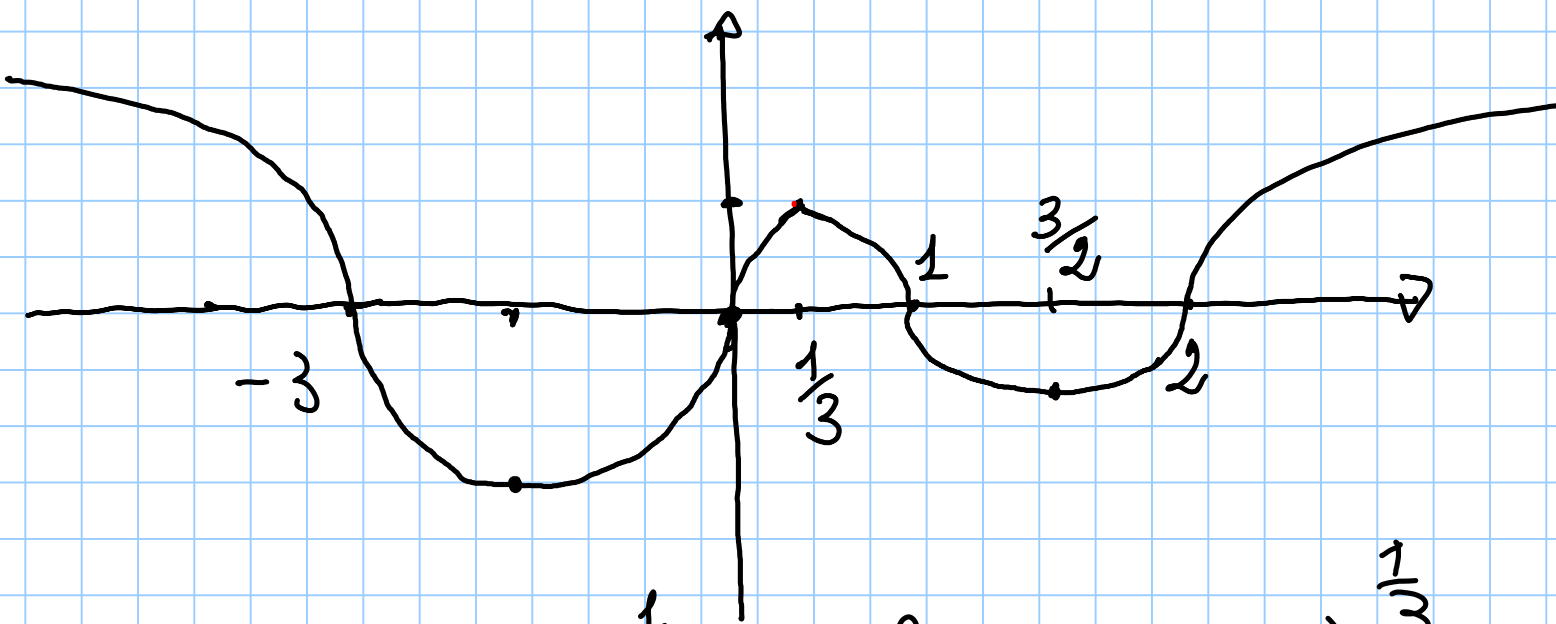
$$x^2 + 3x = x(x+3)$$

f concava in $(-\infty, -3) \cup \left(0, \frac{1}{3}\right)$

convessa in $(-3, 0)$

$$\lim_{x \to (-3)} f'(x) = -\infty \; ; \; x=-3 \text{ Flesso DISCENDENTE}$$

$$\lim_{x \to 0} f'(x) = +\infty \; ; \; x=0 \text{ Flesso ASCENDENTE}$$



$$f\left(\frac{3}{2}\right) = \left(-\frac{1}{4}\right)^{\frac{1}{3}} \qquad f\left(\frac{1}{3}\right) = \left(\frac{10}{9}\right)^{\frac{1}{3}}$$

$$f\left(-\frac{3}{2}\right) = \left(-\frac{9}{4}\right)^{\frac{1}{3}}$$

$$f(x) = x\, e^{\frac{1}{\log(x)}} \quad , \quad x \in (0,+\infty) \setminus \{1\}$$

$$\lim_{x \to +\infty} f(x) = +\infty \; ;$$

$$f(x) = x\, e^{\frac{1}{\log(x)}} = x \left(1 + \frac{1}{\log(x)} + o\left(\frac{1}{\log(x)}\right)\right)$$

$$= x + \frac{x\,(1+o(1))}{\log(x)} \; ;$$

$$f(x) - x = \frac{x}{\log(x)}(1+o(1)) \longrightarrow +\infty$$

NO ASINT. OBLIQUO, $x \to +\infty$

$$\lim_{x \to 1^+} f(x) = 1 \cdot e^{\frac{1}{0^+}} = e^{+\infty} = +\infty$$

$$\lim_{x \to 1^-} f(x) = 1 \cdot e^{1/0^-} = e^{-\infty} = 0^+$$

$x = 1$ ASINTOTO VERTICALE (DESTRO).

$$\lim_{x \to 0^+} f(x) = 0^+ e^{\frac{1}{-\infty}} = 0^+ \cdot 1 = 0^+$$

ESTENDIBILE PER CONTINUITA'

$$f(0) = 0, \quad \lim_{x \to 0^+} f(x) = f(0)$$

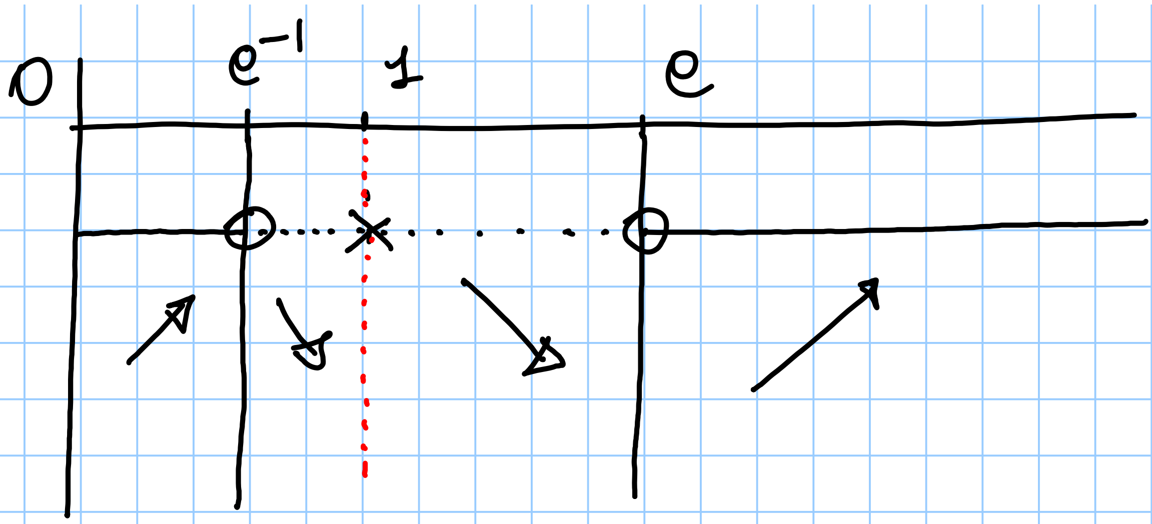
$$f'(x) = e^{\frac{1}{\log(x)}} + x\, e^{\frac{1}{\log(x)}} \left(-\frac{1}{(\log(x))^2} \cdot \frac{1}{x} \right)$$

$$= e^{\frac{1}{\log(x)}} \left[1 - \frac{1}{(\log(x))^2} \right] \geq 0 \quad , \begin{matrix} x \neq 1 \\ x \neq 0 \end{matrix}$$

$$1 - \frac{1}{(\log(x))^2} \geq 0 \quad ; \quad \log^2(x) \geq 1$$

$$\{ \log(x) \leq -1 \} \cup \{ \log(x) \geq 1 \}$$

$$\{ x \leq e^{-1} \} \cup \{ x \geq e \}$$



$x = e^{-1}$ max locale $f(e^{-1}) = \frac{1}{e^2}$

$x = e$ min locale $f(e) = e^2$

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} e^{\frac{1}{\log(x)}}\left(1 - \frac{1}{\log^2(x)}\right) =$$

$$= e^{\frac{1}{-\infty}}\left(1 - \frac{1}{+\infty}\right) = e^{0^-}\cdot(1-0^+) = 1$$

NOTA $\lim_{x\to 0^+} f(x) = f(0)$! $\Rightarrow 0 \notin \mathrm{dom}(f')$

$\lim_{x\to 1^-} f(x) = 0^+$! $\Rightarrow \lim_{x\to 1^-} f'$?

$$\lim_{x\to 1^-} f'(x) = e^{\frac{1}{0^-}}\left(1-\frac{1}{(0^-)^2}\right) =$$

$$= e^{-\infty}\cdot(-\infty) = 0^+\cdot(-\infty)$$

?

$$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} e^{\frac{1}{\log(x)}}\left(1-\frac{1}{(\log(x))^2}\right) =$$

$$= \lim_{\substack{y\to 0^- \\ y=\log(x)}} e^{\frac{1}{y}}\left(1-\frac{1}{y^2}\right) = \lim_{y\to 0^-} \frac{e^{\frac{1}{y}}}{y^2}(-1+o(1)) =$$

$$= \lim_{t\to+\infty} -t^2 e^{-t} = \lim_{t\to+\infty} -\frac{t^2}{e^t} = 0^-$$

$t=-\frac{1}{y} \longrightarrow +\infty,\ y\to 0^-$

$$f''(x) = \frac{d}{dx}\left(e^{\frac{1}{\log(x)}}\left(1-\frac{1}{\log^2(x)}\right)\right) =$$

$$= e^{\frac{1}{\log(x)}}\left(-\frac{1}{\log^2(x)}\cdot\frac{1}{x}\right)\left(1-\frac{1}{\log^2(x)}\right)+$$

$$e^{\frac{1}{\log(x)}}\frac{2}{(\log(x))^3}\cdot\frac{1}{x}=$$

$$\frac{e^{\frac{1}{\log(x)}}}{(\log(x))^2\cdot x}\left[-1+\frac{1}{\log^2(x)}+\frac{2}{\log(x)}\right]$$

$$f''(x)\geq 0 \iff -1+\frac{1}{\log^2(x)}+\frac{2}{\log(x)}\geq 0$$

$$x\neq 0,1$$

$$-\log^2(x)+2\log(x)+1\geq 0\,,\quad \log(x)=y$$

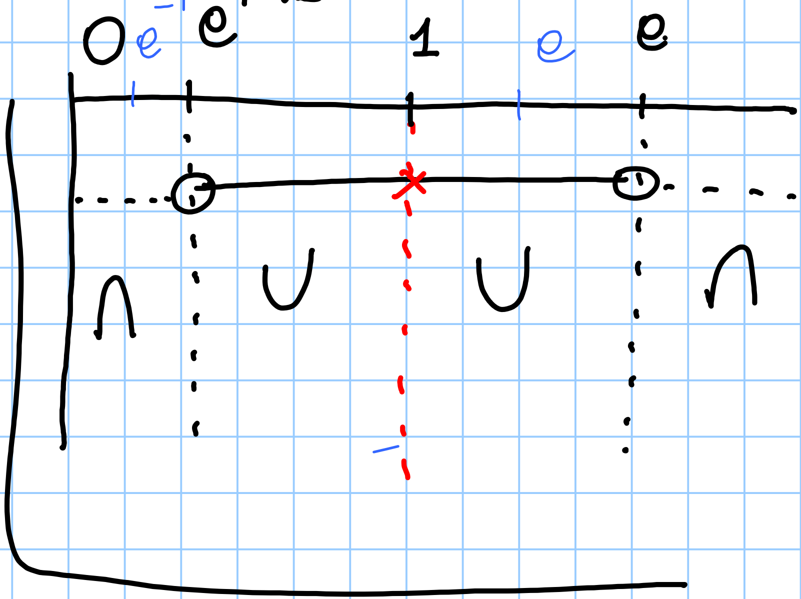
$$-y^2+2y+1\geq 0\,,\quad y_\pm=\frac{-1\pm\sqrt{1+1}}{-1}=$$

$$=1\pm\sqrt{2}$$

$$1-\sqrt{2}<y<1+\sqrt{2}$$

$$1-\sqrt{2} < \log(x) < 1+\sqrt{2}$$

$$e^{1-\sqrt{2}} < x < e^{1+\sqrt{2}}$$



$$X_0 = e^{1-\sqrt{2}}$$

$$f'\left(e^{1-\sqrt{2}}\right) < 0$$

$$e^{1-\sqrt{2}} > e^{-1}$$

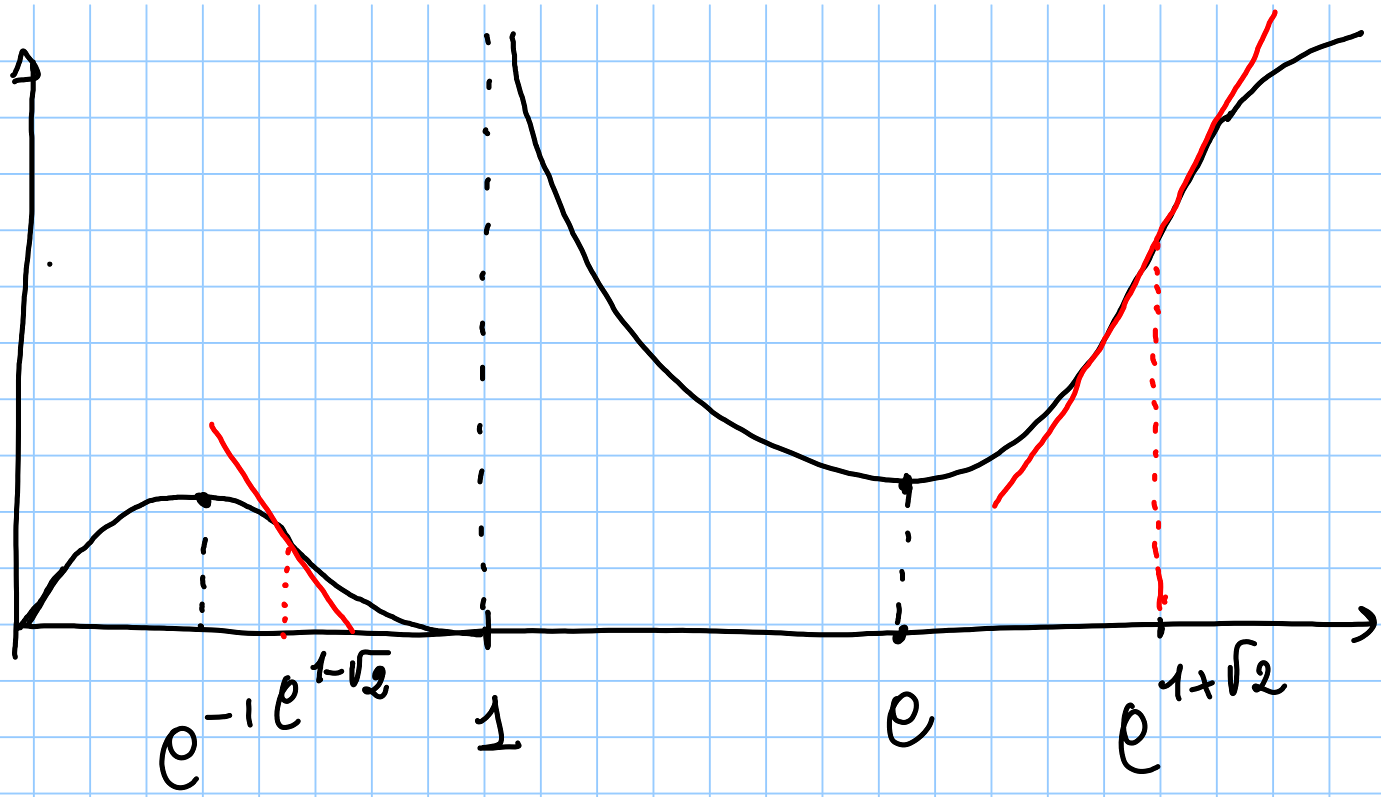
$X_0 = e^{1-\sqrt{2}}$ FLESSO DISCENDENTE

$$X_0 = e^{1+\sqrt{2}}$$

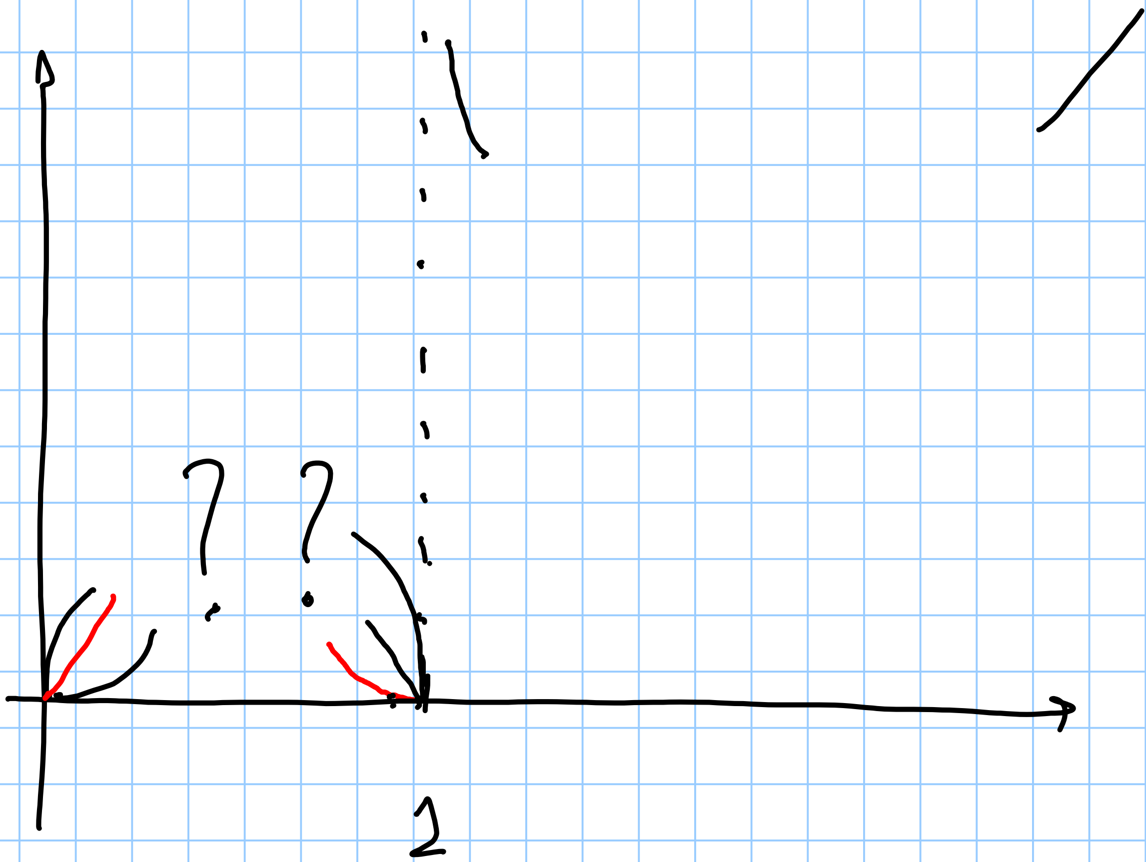
$$f'\left(e^{1+\sqrt{2}}\right) > 0$$

$$e^{1+\sqrt{2}} > e$$

$X_0 = e^{1+\sqrt{2}}$ FLESSO ASCENDENTE



e^{-1}
$e^{1-\sqrt{2}}$
1
e
$e^{1+\sqrt{2}}$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