ALTRA DIMOSTRAZIONE DEL FATTO CHE

$$0,\overline{9} = 1$$

$$\sum_{k=1}^{N} x^k = \frac{x - x^{N+1}}{1-x}$$

$$0,\overline{9} = \lim_{N \to +\infty} 0,\overset{1}{9}\,\overset{2}{9}\,\overset{3}{9}\ldots\overset{N-1}{9}\,\overset{N}{9}$$

$$0,\underset{1}{9}\,\underset{2}{9}\,\underset{3}{9}\ldots\underset{N-1}{9}\,\underset{N}{9} = \sum_{k=1}^{N} 9\cdot 10^{-k} =$$

$$9\cdot\sum_{k=1}^{N} 10^{-k} \underset{x=10^{-1}}{=} 9\cdot\frac{10^{-1}-10^{-N-1}}{1-10^{-1}} =$$

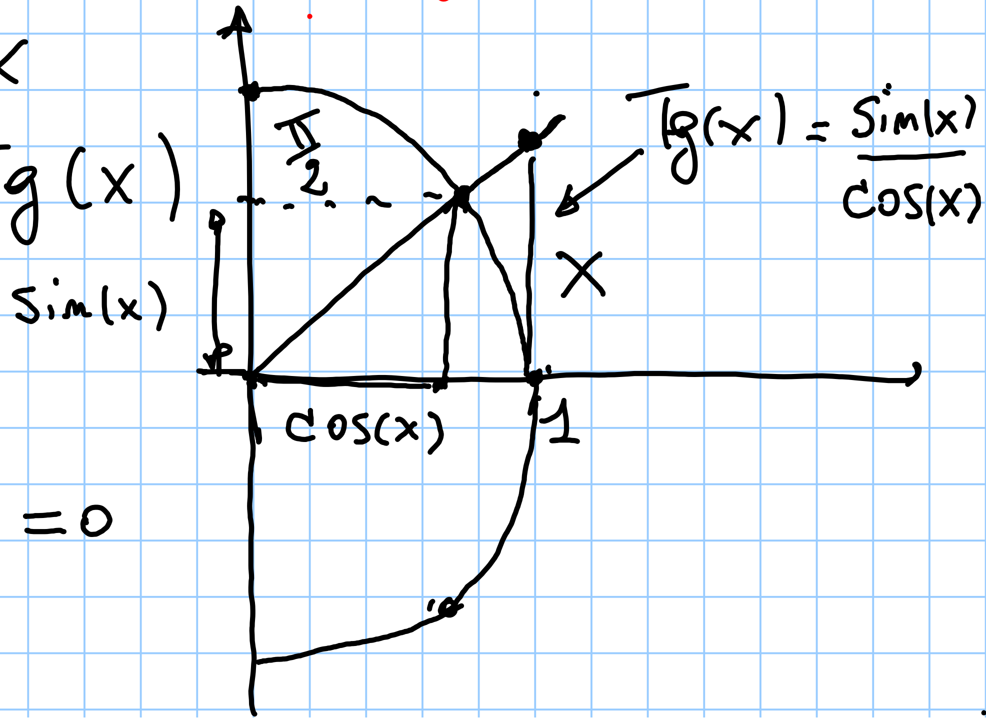
$$= 9 \cdot \frac{1}{10} \cdot \frac{1 - 10^{-N}}{9/10} = 1 - 10^{-N}$$

$$0,\overline{9} = \lim_{N \to +\infty} 1 - 10^{-N} = 1$$

CONTINUITA' di $\sin(x)$, $\cos(x)$, $Tg(x)$

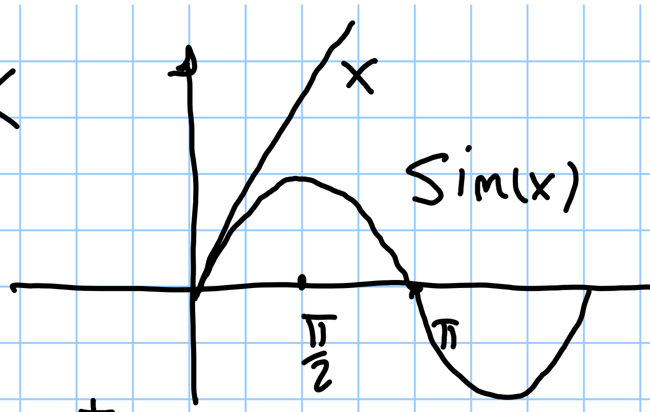
$\sin(x) < x$

$x < Tg(x)$



$$\lim_{x \to 0} \sin(x) = 0$$

$x > 0$, $\quad \text{sim}(x) < x$



$$0 < \text{sim}(x) < x \longrightarrow 0^+$$

$$x \longrightarrow 0^+$$

$$\lim_{x \to 0^+} \text{sim}(x) = 0^+$$

$y = -x$, $\quad x > 0$

$$\text{sim}(y) = \text{Sim}(-x) = -\text{sim}(x)$$

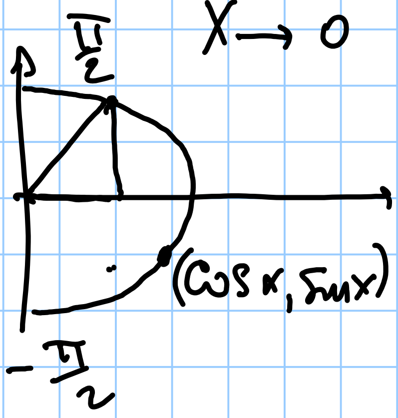
$$\lim_{y \to 0^-} \text{Sim}(y) = \lim_{x \to 0^+} -\text{Sim}(x) = 0^-$$

$$\lim_{y \to 0^-} \text{Sim}(y) = 0^-$$

$$\lim_{x \to 0^\pm} \text{sin}(x) = 0 \Longrightarrow \lim_{x \to 0} \text{Sin}(x) = 0$$

$$\lim_{x\to 0}\cos(x) = \lim_{x\to 0}\sqrt{1-\sin^2(x)} =$$

$y = \sin(x) \longrightarrow 0$



$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$= \lim_{y\to 0}\sqrt{1-y^2} = 1$$

$$\lim_{x\to x_0}\sin(x) = \sin(x_0)$$

$$x = x_0 + y \quad , \quad y \to 0$$

$$\sin(x) = \sin(x_0+y) =$$

$$= \sin(x_0)\cos(y) + \sin(y)\cos(x_0)$$

$$\lim_{x\to x_0}\sin(x) = \lim_{y\to 0}\sin(x_0+y) = \sin(x_0)\,1 + 0\cdot\cos(x_0)$$

$$= \sin(x_0)$$

Per $\cos(x)$ si procede analogamente.

$$Tg(x) = \frac{\sin(x)}{\cos(x)} \qquad x \neq \frac{\pi}{2} + k\pi$$
$$k \in \mathbb{Z}$$

La continuità di $Tg(x)$ segue dalla continuità di $\sin(x)$, $\cos(x)$ e dalla continuità del RAPPORTO di funzioni continue

LIMITI NOTEVOLI

$$\boxed{\lim_{x\to 0} \frac{\sin(x)}{x} = 1^-}$$

$\otimes$ $\sin(x) < x$ $\quad x > 0$

$\otimes$ $x < \mathrm{Tg}(x)$ $\quad x \in \left(0, \frac{\pi}{2}\right)$

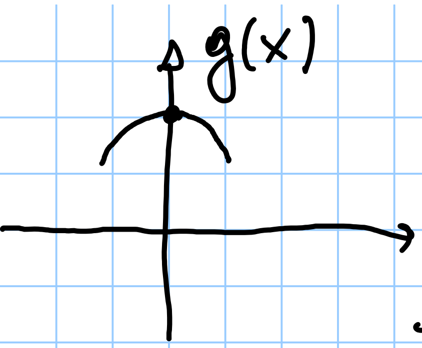
$$\underbrace{\cos(x)}_{\substack{\downarrow\ x\to 0^+ \\ 1}} < \underbrace{\frac{\sin(x)}{x}}_{\substack{\downarrow \\ 1\ \ x\to 0^+}} < \underbrace{1}_{\substack{\downarrow\ 1 \\ x\to 0^+}} \qquad \boxed{x < \frac{\sin(x)}{\cos(x)}}$$

$\frac{\sin(x)}{x}$ è PARi $\Longrightarrow$

$$\lim_{x\to 0^-} \frac{\sin(x)}{x} = \lim_{y\to 0^+} \frac{\sin(-y)}{-y} = \lim_{y\to 0^+} \frac{\sin(y)}{y}$$

$x = -y$ $\qquad = 1^-$

$$f(x) = \frac{\sin x}{x}, \qquad x \neq 0$$



$$g(x) = \begin{cases} \frac{\sin(x)}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

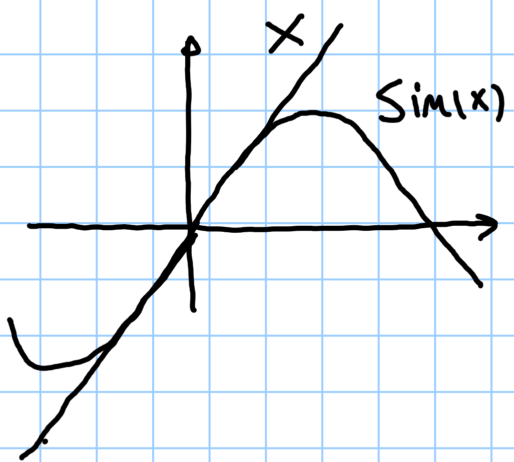
è continua in $\mathbb{R}$ $\qquad g(0) = 1$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1 \iff$$

$$\lim_{x \to 0} \left(\frac{\sin(x)}{x} - 1 \right) = 0$$

$$\lim_{x \to 0} \frac{\sin(x) - x}{x} = 0$$

$$\sin(x) - x = o(x), \quad x \to 0$$

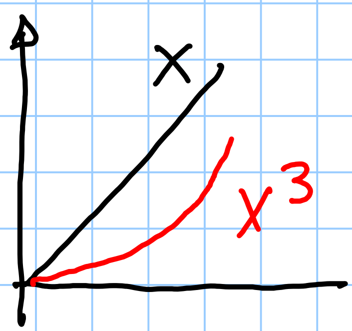


$$\underline{\sin(x) = x + o(x), \quad x \to 0}$$

$f(x) \to 0$, $g(x) \to 0$, $x \to x_0$, se $\boxed{\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0}$

SI DICE CHE f è UN INFINITESIMO di ordine SUPERIORE a g per $x \to x_0$

$$\boxed{f(x) = o(g(x)), \quad x \to x_0}$$

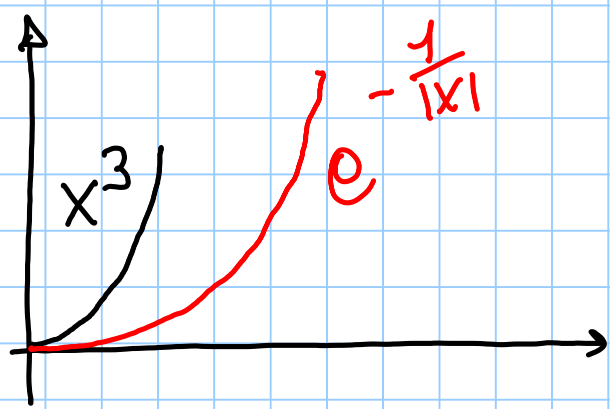


$f(x) = x^3$

$g(x) = x$

$f(x) = o(g(x))$, $x \to 0$

$x^3 = o(x)$, $x \to 0$



$f(x) = e^{-\frac{1}{|x|}}$, $x \to 0$

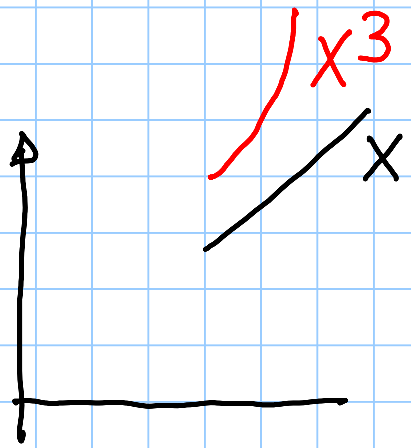
$g(x) = x^3$,

$f(x) = o(g(x))$

$f(x) \to +\infty$,
$g(x) \to +\infty$, $x \to x_0$, se $\boxed{\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0}$

SI DICE CHE f è UN INFINITO di ordine INFERIORE a g per $x \to x_0$

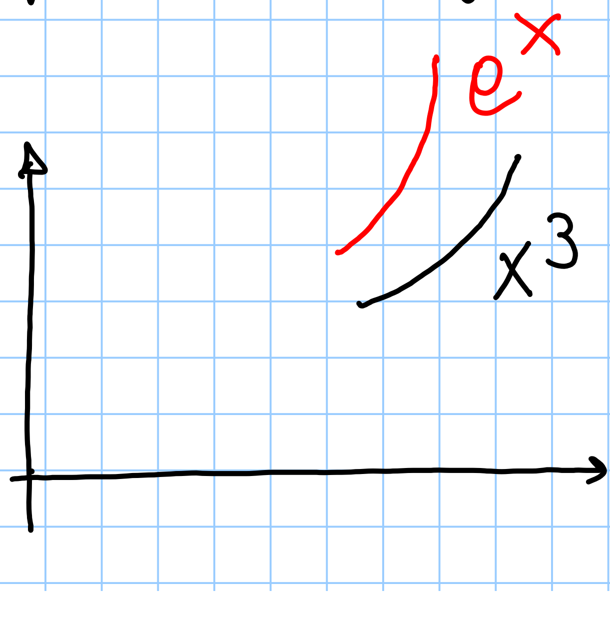
$$\boxed{f(x) = o(g(x)), \quad x \to x_0}$$



$f(x) = x$
$g(x) = x^3$

$f(x) = o(g(x))$
$x \to +\infty$

$x^3 = o(x)$, $x \to +\infty$



$f(x) = x^3$,
$g(x) = e^x$, $x \to +\infty$

$f(x) = o(g(x))$

RICORDARE:

SE $f(x) \to 0$, $g(x) \to 0$

o SE $f(x) \to \pm\infty$, $g(x) \to \pm\infty$

per $x \to x_0$

e se $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$

ALLORA "$f(x)$ è MOLTO più piccolo di $g(x)$" definitivamente per $x \to x_0$

$$\lim_{x\to 0} \frac{\pi x^3 + x^5}{x^5 + x^6} \sin(x^2) =$$

$$\lim_{x\to 0} \frac{\pi x^3 + x^5}{x^5 + x^6}\, x^2 \cdot \frac{\sin(x^2)}{x^2} =$$

$$= \lim_{x\to 0} \frac{\pi \cancel{x^3}}{\cancel{x^5}} \cdot \frac{\left(1 + \frac{x^5}{\pi x^3}\right)}{\left(1 + \frac{x^6}{x^5}\right)}\, \cancel{x^2}\, \frac{\sin(x^2)}{x^2}$$

$$= \lim_{x\to 0} \pi\, \frac{(1 + o(1))}{(1 + o(1))} \cdot \frac{\sin(x^2)}{x^2} = \pi \cdot 1 \cdot 1 = \pi$$

$$\sin y = y + o(y), \quad y \to 0$$

$$\Downarrow$$

$$y = x^2 \qquad \sin(x^2) = x^2 + o(x^2), \quad x \to 0$$

$$\lim_{x\to 0} \frac{\pi x^3 + x^5}{x^5 + x^6} \cdot \sin(x^2) =$$

$$= \lim_{x\to 0} \frac{\pi x^3 + x^5}{x^5 + x^6} \cdot \left(x^2 + o(x^2)\right) =$$

$$= \lim_{x\to 0} \frac{\pi x^3}{x^5} \frac{(1+o(1))}{(1+o(1))} \cdot x^2 (1+o(1)) = \pi$$

ABBIAMO USATO IL FATTO CHE:

$$x^2 + o(x^2) = x^2 \left(1 + \frac{o(x^2)}{x^2}\right) = x^2 (1+o(1))$$

IN PARTICOLARE SI HA PER DEFINIZIONE CHE SE $f(x) \to 0$ ALLORA $\frac{o(f(x))}{f(x)} = o(1)$

$$\boxed{\lim_{x\to 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}}$$

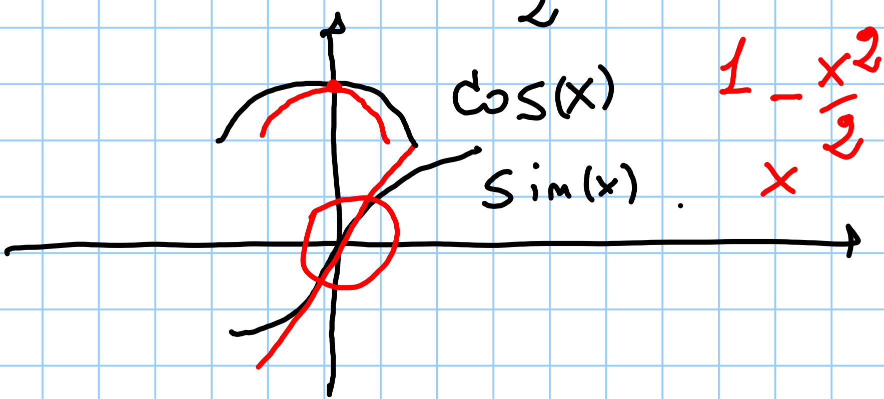
$$\frac{1-\cos(x)}{x^2} = 2\,\frac{\sin^2\left(\frac{x}{2}\right)}{x^2} = \frac{2}{4}\,\frac{\sin^2\left(\frac{x}{2}\right)}{\frac{x^2}{4}} =$$

$$= \frac{1}{2}\left(\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\right)^2 \xrightarrow[x\to 0]{} \frac{1}{2}\cdot 1^2 = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{1-\cos(x)-\frac{x^2}{2}}{x^2} = 0$$

$$1-\cos(x)-\frac{x^2}{2} = o(x^2)$$

$$\cos(x) = 1-\frac{x^2}{2}+o(x^2), \quad x\to 0$$



$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to -\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\boxed{\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e}$$

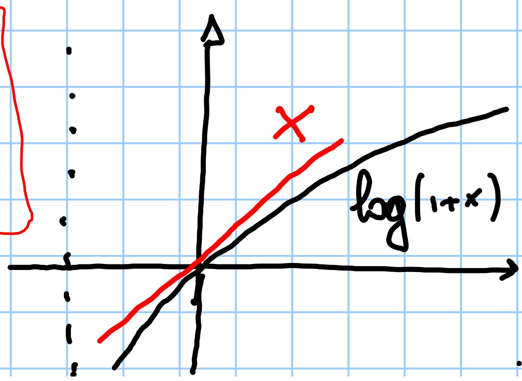
$$\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = \lim_{y \to +\infty} \left(1 + \frac{1}{y}\right)^y = e$$

$$\frac{1}{x} = y \xrightarrow[x \to 0^+]{} +\infty$$

$$\lim_{x \to 0^-} (1+x)^{\frac{1}{x}} = \lim_{y \to -\infty} \left(1 + \frac{1}{y}\right)^y = e$$

$$\frac{1}{x} = y \xrightarrow[x \to 0^-]{} -\infty$$

$$\boxed{\lim_{x \to 0} \frac{\log(1+x)}{x} = 1}$$



$$\frac{\log(1+x)}{x} = \frac{1}{x}\log(1+x) = \log(1+x)^{\frac{1}{x}}$$

$$\xrightarrow[x\to 0]{} \log(e) = 1$$

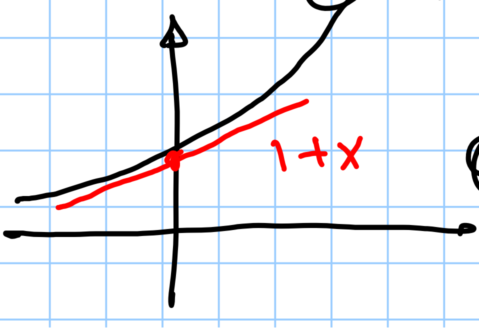
$$\log(1+x) = x + o(x), \quad x \to 0$$

$$\boxed{\lim_{x\to 0} \frac{e^x - 1}{x} = 1}$$

$$e^x - 1 = y \xrightarrow[x\to 0]{} 0$$

$$\lim_{y\to 0} \frac{y}{\log(1+y)} = \lim_{y\to 0} \frac{1}{\frac{\log(1+y)}{y}} = 1$$

$$e^x = 1 + y \qquad x = \log(1+y)$$



$$e^x = 1 + x + o(x), \quad x \to 0$$

$$\boxed{\lim_{x\to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha}$$

$$\alpha = 0 \quad \lim_{x\to 0} \frac{1-1}{x} = \lim_{x\to 0} \frac{0}{x} = 0$$

$\alpha \neq 0$

$$\frac{(1+x)^\alpha - 1}{x} = \frac{(1+x)^\alpha - 1}{x} \, \frac{\log(1+x)}{\log(1+x)} =$$

$$= \frac{(1+x)^\alpha - 1}{\log(1+x)} \cdot \underbrace{\frac{\log(1+x)}{x}}_{\to 1}$$

$$t = \log(1+x) \longrightarrow 0$$

$$e^t = 1+x$$

$$\frac{(1+x)^\alpha - 1}{\log(1+x)} = \frac{e^{\alpha t} - 1}{t} = \frac{e^{\alpha t} - 1}{\alpha t} \cdot \alpha$$

$$\underset{\alpha t = y \longrightarrow 0,\ t \to 0}{=} \frac{e^y - 1}{y} \cdot \alpha \xrightarrow[y \to 0]{} \alpha$$

- $$\lim_{x\to 0} \frac{\operatorname{Tg} x}{x} = \lim_{x\to 0} \frac{\sin(x)}{x} \cdot \frac{1}{\cos(x)} = 1$$

- $$\lim_{x\to 1} \frac{x^\alpha - 1}{x^\beta - 1} = \frac{\alpha}{\beta}, \quad \beta \neq 0$$

$\alpha \neq 0$

$$\frac{x^\alpha - 1}{x^\beta - 1} = \frac{(1+y)^\alpha - 1}{(1+y)^\beta - 1} =$$

$$x = 1+y, \quad y = x-1 \xrightarrow[x\to 1]{} 0,$$

$$\overset{y\to 0}{=} \frac{\cancel{1} + \alpha y + o(y) - \cancel{1}}{\cancel{1} + \beta y + o(y) - \cancel{1}} = \frac{\alpha \cancel{y}}{\beta \cancel{y}} \cdot \frac{1+o(1)}{1+o(1)}$$

$$\xrightarrow[y\to 0]{} \alpha/\beta$$

$$f(x) = \frac{1-x^{N+1}}{1-x}$$

$$\lim_{x\to 1} f(x) = N+1$$

$$\lim_{x\to -\infty} \frac{(4-e^x)^{\pi}-4^{\pi}}{e^{2x}}$$

$$\boxed{(1+z)^{\alpha} = 1+\alpha z + o(z),\ z\to 0}$$

$$\frac{(4-e^x)^{\pi}-4^{\pi}}{e^{2x}} = \frac{(4-y)^{\pi}-4^{\pi}}{y^2} =$$

$y = e^x \to 0^+,\ x\to -\infty$

$$\frac{4^{\pi}\left(1-\frac{y}{4}\right)^{\pi}-4^{\pi}}{y^2} = 4^{\pi}\cdot\frac{\left(1-\frac{y}{4}\right)^{\pi}-1}{y^2},$$

$z = -\frac{y}{4}$

$$\underset{y\to 0^+}{=} 4^{\pi}\cdot\frac{\cancel{1}+\pi\left(-\frac{y}{4}\right)+o(y)-\cancel{1}}{y^2} =$$

$$= 4^{\pi}\cdot\left(-\frac{\pi}{4}\right)\cdot\frac{y+o(y)}{y^2} = -4^{\pi-1}\pi\,\frac{1+o(1)}{y}$$

$$\xrightarrow[y\to 0^+]{} -\frac{4^{\pi-1}\pi}{0^+} = -\infty$$

$$\lim_{n} \frac{n^4 + e^{-3n} - \log\left(1+e^{n^4}\right) + n^3 \sin(n^2) e^{-5n}}{e^{-5n}\sin(n) - e^{n} - e^{-n^2}\cos(n^5) + \sqrt[3]{e^{3n} - e^{-n}}}$$

(1)

$$\log\left(1+e^{n^4}\right) = \log\left(e^{n^4}\left(1+e^{-n^4}\right)\right)$$

$$= \underbrace{n^4} + \log\left(1+\underbrace{e^{-n^4}}_{\to 0}\right)$$

$$\log(1+x) = x + o(x), \quad x \to 0$$

$$\log\left(1+e^{-n^4}\right) = e^{-n^4} + o\left(e^{-n^4}\right) \quad n \to +\infty$$

$$n^4 + e^{-3n} - \log\left(1+e^{n^4}\right) + n^3 \sin(n^2) e^{-5n}$$

$$= \cancel{n^4} + e^{-3n} - \cancel{n^4} - e^{-n^4} + o\left(e^{-n^4}\right) + n^3 \sin(n^2) e^{-5n}$$

$$e^{-3n} + e^{-5n} \cdot n^3$$

$$e^{-3n}\left(1 + \cancel{\frac{n^3}{e^{2n}}}\right) = e^{-3n}\left(1+o(1)\right)$$

$$e^{-3n} - e^{-n^4} =$$

$$e^{-3n}\left(1 - e^{-n^4+3n}\right)$$

$$e^{-3n}\left(1 + o(1)\right)$$

$$N(n) = e^{-3n}\left(1 + o(1)\right)$$

$$D(n) = e^{-3n}\left(-\frac{1}{3} + o(1)\right)$$

Ex

$$\lim_{n} \frac{N(n)}{D(n)} = \lim_{n} \frac{e^{-3n}(1+o(1))}{e^{-3n}\left(-\frac{1}{3}+o(1)\right)} = -3$$

$$(e^{-2n}) \qquad 0^-$$

$$\sqrt[3]{e^{3n} - e^{-n}} =$$

$$= e^{n}\sqrt[3]{1 - e^{-4n}} =$$

$$= e^{n}\left(1-\frac{1}{3}e^{-4n}+o\left(e^{-4n}\right)\right) =$$

$$= e^{n}-\frac{1}{3}e^{-3n}+o\left(e^{-3n}\right)$$

IMPORTANTE :

- $$\lim_{x\to 0^+} x\log x \underset{y=\frac{1}{x}\xrightarrow[x\to 0^+]{}+\infty}{=} \lim_{y\to+\infty} -\frac{\log(y)}{y} = 0^-$$

- $$\lim_{x\to 0^-} \frac{e^{\frac{1}{x}}}{x} \underset{y=-\frac{1}{x}\xrightarrow[x\to 0^-]{}+\infty}{=} \lim_{y\to+\infty} -ye^{-y} = \lim_{y\to+\infty} -\frac{y}{e^{y}} = 0^-$$

Ex. $$\lim_{x\to 0} \frac{e^{-\frac{1}{|x|}}}{x} = 0 \; ; \; \lim_{x\to 0} x\log|x| = 0$$

$$f(x) = \frac{\log(x)}{\sqrt{|x^2 - x|}}$$

$$x > 0 \; ; \quad x^2 - x \neq 0$$

$$\mathrm{dom}(f) = (0, +\infty) \setminus \{1\}$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\log(x)}{\sqrt{|x|\,\underbrace{|x-1|}_{= 1 + o(1)}}} =$$

$$= \frac{-\infty}{0^+} = -\infty$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{\log(x)}{\sqrt{|x(x-1)|}} =$$

$$= \lim_{x \to 1} \frac{1}{\sqrt{|x|}} \frac{\log(x)}{\sqrt{|x-1|}} = \left\{ \begin{array}{c} y = x-1 \to 0 \\ x \to 1 \end{array} \right\}$$

$$= \lim_{y \to 0} \frac{1}{1+o(1)} \cdot \frac{\log(1+y)}{\sqrt{|y|}} =$$

limite del prodotto = prodotto dei limiti

SE NON CI SONO F.I.

$$= \lim_{y \to 0} \frac{y + o(y)}{|y|^{\frac{1}{2}}} = \lim_{y \to 0} \frac{y}{|y|^{\frac{1}{2}}} (1 + o(1))$$

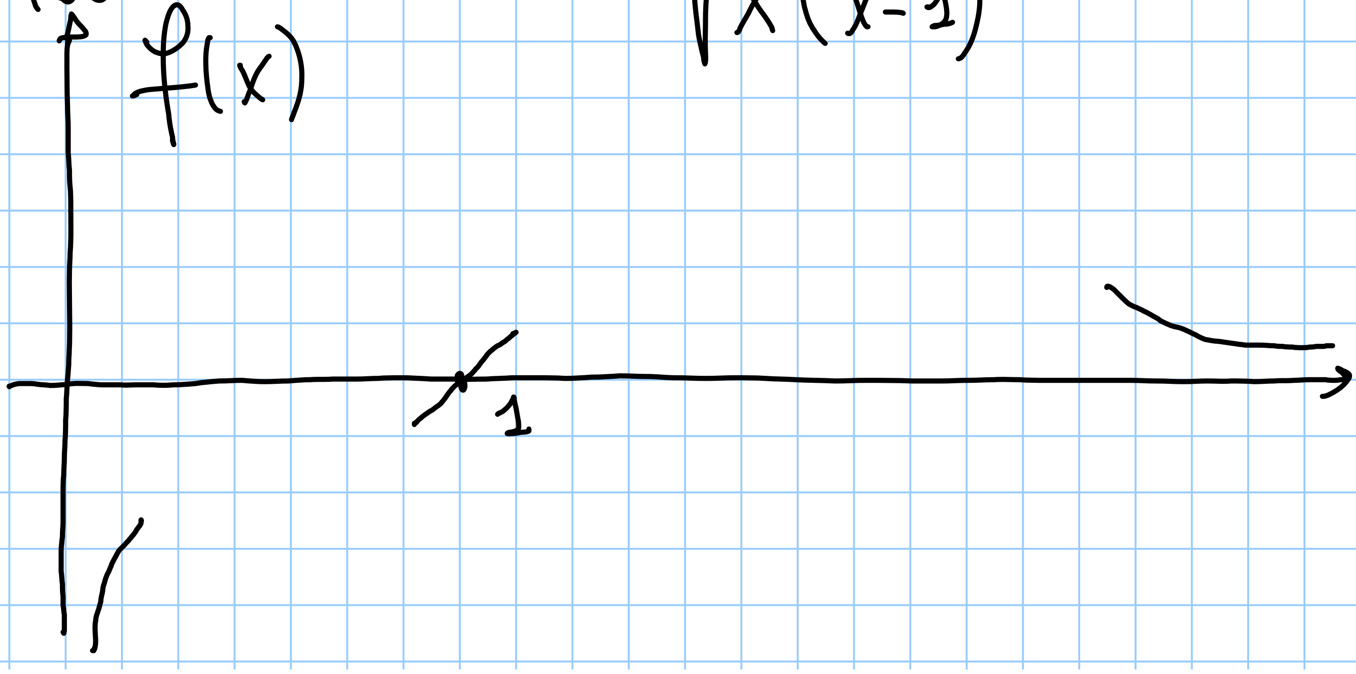
$$= \lim_{y \to 0} \operatorname{sgn}(y) \frac{|y|}{|y|^{\frac{1}{2}}} = 0$$

$$= \begin{cases} \text{Se } y \to 0^+ & \dfrac{|y|}{|y|^{\frac{1}{2}}} = |y|^{\frac{1}{2}} \to 0, \ y \to 0^+ \\ \text{Se } y \to 0^- & -\dfrac{|y|}{|y|^{\frac{1}{2}}} = -|y|^{\frac{1}{2}} \to 0, \ y \to 0^- \end{cases}$$

$$x \to 1^{\pm} \iff y = x - 1 \to 0^{\pm}$$

$$\lim_{x \to 1^{\pm}} f(x) = 0^{\pm}$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\log(x)}{\sqrt{x(x-1)}} = 0^+$$



$$f(x) = \frac{e^{\frac{x^2}{x^2-4}}}{\sqrt{|x^2-x-2|}} = \frac{e^{\frac{x^2}{(x-2)(x+2)}}}{\sqrt{|(x+1)(x-2)|}}$$

$$x^2 - x - 2 = (x-2)(x+1)$$

$$x^2 - 4 = (x-2)(x+2)$$

$$\operatorname{dom}(f) = \mathbb{R} \setminus \{-2, -1, 2\}$$

$$\frac{x^2}{x^2-4} = \frac{x^2}{(x-2)(x+2)} \overset{x\to 2}{=} \frac{4+o(1)}{(4+o(1))(x-2)} =$$

$$= \frac{1+o(1)}{x-2}, \quad x \to 2$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{e^{\frac{1+o(1)}{x-2}}}{\sqrt{(x+1)(x-2)}} =$$

$$= \frac{e^{1/0^+}}{\sqrt{3 \cdot 0^+}} = \frac{e^{+\infty}}{0^+} = +\infty$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{e^{\frac{1+o(1)}{x-2}}}{\sqrt{(x+1)|x-2|}} =$$

$\to e^{-\infty} = 0$

$\to 0$

$$\lim_{x\to 2^-} \frac{e^{\frac{1+o(1)}{x-2}}}{\sqrt{(3+o(1))|x-2|}} \underset{x-2=t}{=} \lim_{t\to 0^-} \frac{e^{\frac{1+o(1)}{t}}}{\sqrt{3|t|\cdot(1+o(1))}} =$$

$$\left\{ \frac{1}{\sqrt{3+o(1)}} = \frac{1}{\sqrt{3(1+o(1))}} = \frac{1}{\sqrt{3}}\,\frac{1}{\sqrt{1+o(1)}} = \frac{1+o(1)}{\sqrt{3}} \right\}$$

$$= \lim_{t\to 0^-} \frac{e^{\frac{1+o(1)}{t}}}{\sqrt{|t|}}\,\frac{1+o(1)}{\sqrt{3}}$$

$$\lim_{t\to 0^-} \frac{\overset{0}{e^{\frac{1+o(1)}{t}}}}{\underbrace{\sqrt{|t|}}_{\to 0}} \, \frac{(1+o(1))}{\sqrt{3}} =$$

$$\frac{1}{\sqrt{3}} \lim_{t\to 0^-} e^{\frac{1+o(1)}{t} - \log\sqrt{|t|}} = \circledast$$

$$= \frac{1}{\sqrt{3}} \lim_{t\to 0^-} e^{\frac{1}{t}\left(1+o(1) - t\log\sqrt{|t|}\right)} =$$

$$= \frac{1}{\sqrt{3}} \lim_{t \to 0^-} e^{\frac{1}{t}(1+o(1))} = \frac{1}{\sqrt{3}} e^{-\infty} = 0^+$$

$$\lim_{x \to 2^-} f(x) = 0^+$$

$\circledast$

$$\lim_{t \to 0^-} \Big(\overbrace{\frac{1+o(1)}{t}}^{-\infty} - \overbrace{\log\sqrt{1+t}}^{-\infty} \Big)$$

Qual è il TERMINE DOMINANTE?

$$\lim_{t \to 0^-} \frac{\frac{1+o(1)}{t}}{\log\sqrt{1+t}} = \lim_{t \to 0^-} \frac{1+o(1)}{t\log\sqrt{1+t}} =$$

$$= \left\{ y = \frac{1}{t} \to -\infty, \ t \to 0^- \right\} =$$

$$= \lim_{y \to -\infty} \frac{y\,(1+o(1))}{\log\left(|y|^{-\frac{1}{2}}\right)} =$$

$$= \lim_{y \to -\infty} \frac{y}{-\frac{1}{2}\log|y|} = +\infty$$

$\frac{1}{t}$ E' IL TERMINE DOMINANTE

Se avessimo calcolato

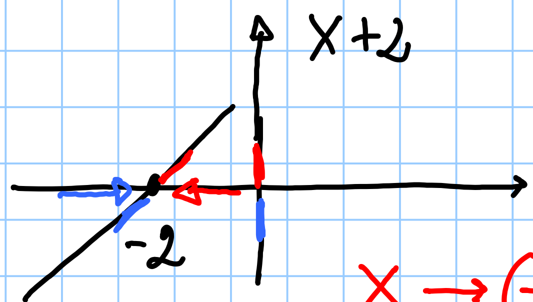
$$\lim_{t \to 0^-} \frac{\log\left(\sqrt{|t|}\right)}{\frac{1+o(1)}{t}} = 0 \Rightarrow \text{"}\log\left(\sqrt{|t|}\right)$$

è più piccolo" $\Rightarrow \frac{1}{t}$ è il Termine dominante

$$\frac{x^2}{x^2-4} = \frac{x^2}{(x-2)(x+2)} \underset{x\to(-2)^+}{=} \frac{4+o(1)}{((-4)+o(1))(x+2)} =$$

$$= \frac{-1+o(1)}{x+2} \xrightarrow[x\to(-2)^+]{} \frac{-1}{0^+} = -\infty$$

$$\frac{x^2}{x^2-4} \underset{x\to(-2)^-}{=} \cdots = \frac{-1+o(1)}{x+2} \xrightarrow[x\to(-2)^-]{} \frac{-1}{0^-} = +\infty$$



$x \to (-2)^+$

$x+2 \to 0^+$

$x \to (-2)^-$

$x+2 \to 0^-$

$$f(x) = \frac{e^{\frac{x^2}{x^2-4}}}{\sqrt{|(x-2)(x+1)|}} \xrightarrow{x \to (-2)^+}$$

$$\xrightarrow[x \to (-2)^+]{} \frac{e^{-1/0^+}}{\sqrt{|(-4)\cdot(-1)|}} = \frac{e^{-\infty}}{2} = 0^+$$

$$\lim_{x \to (-2)^+} f(x) = 0^+$$

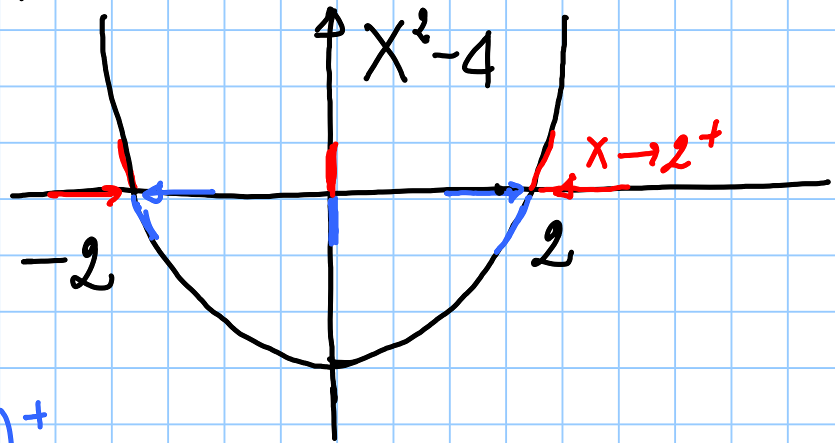
$$\lim_{x \to (-2)^-} f(x) = \dots = \frac{e^{+\infty}}{4} = +\infty$$

ALTERNATIVAMENTE

x^2-4

$x^2-4 \to 0^+$ $\quad x \to (-2)^-$

$x^2-4 \to 0^-$ $\quad x \to (-2)^+$



$$f(x) = \frac{e^{\frac{x^2}{x^2-4}}}{\sqrt{|(x-2)(x+1)|}} \xrightarrow{x\to(-2)^+} \frac{e^{\frac{4}{0^-}}}{\sqrt{|(-4)(-1)|}}$$

$$= \frac{e^{-\infty}}{2} = 0^+$$

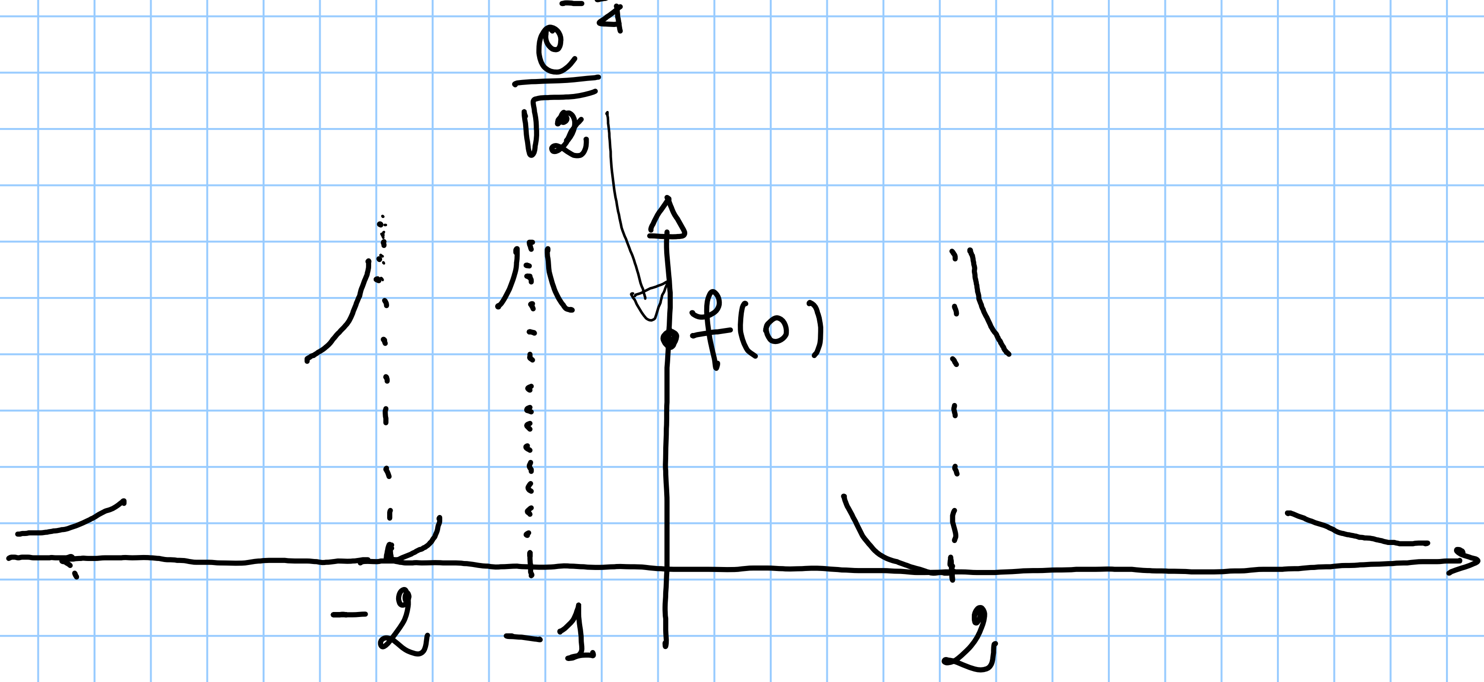
$$f(x) = \ldots \xrightarrow{x\to(-2)^-} \frac{e^{\frac{4}{0^+}}}{\sqrt{(-4)(-1)}} = +\infty$$

$$\lim_{x \to \pm\infty} f(x) = \frac{e^{1}}{+\infty} = 0^{+}$$

$$\lim_{x \to -1^{\pm}} \frac{e^{\frac{x^2}{x^2-4}}}{\sqrt{|(x-2)(x+1)|}} = \frac{e^{\frac{1}{-3}}}{\sqrt{|(-1)\,0^{\pm}|}} =$$

$$= \frac{1}{\sqrt[3]{e}} \cdot \frac{1}{0^{+}} = +\infty$$

f ha ASINTOTI VERTICALI in $-2, -1, 2$



CONFRONTO TRA INFINITI / INFINITESIMI

Se $f(x) \underset{x \to x_0}{\longrightarrow} 0$ e $g(x) \underset{x \to x_0}{\longrightarrow} 0$

$[\ x_0 \in \overline{\mathbb{R}}\ ]$ e se $\displaystyle\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$,

f è un **INFINITESIMO di ORDINE SUPERIORE** rispetto a g per $x \to x_0$.

$$f(x) = o(g(x)), \quad x \to x_0$$

ES. $f(x) = x^5$, $g(x) = x^2$

per $x \to 0$

$\lim_{x \to 0} \frac{x^5}{x^2} = 0$

$f(x) = e^{-\frac{1}{|x|}}$, $g(x) = x$

per $x \to 0$

$\lim_{x \to 0} \frac{e^{-\frac{1}{|x|}}}{x} = 0$

$f(x) = x$, $g(x) = \frac{1}{\log|x|}$

per $x \to 0$

$\lim_{x \to 0} \frac{x}{\frac{1}{\log|x|}} = 0$

In particolare f è MOLTO PIÙ PICCOLO di g per $x \to 0$

ANALOGAMENTE

$f(x) = (x+4)^5$, $g(x) = (x+4)^2$

per $x \to -4$

$$f(x) = (x-x_0)^5, \quad g(x) = (x-x_0)^2$$

per $x \to x_0$, $x_0 \in \mathbb{R}$

$$f(x) = e^{-\frac{1}{|x-x_0|}}, \quad g(x) = x - x_0$$

per $x \to x_0$, $x_0 \in \mathbb{R}$

$$f(x) = x - x_0, \quad g(x) = \frac{1}{\log|x-x_0|}$$

per $x \to x_0$, $x_0 \in \mathbb{R}$

$$f(x) = \frac{1}{x^5}, \quad g(x) = \frac{1}{x^2}$$

per $x \to +\infty$

$$f(x) = e^{-x}, \quad g(x) = \frac{1}{x^4}$$

per $x \to +\infty$

$$f(x) = e^x, \quad g(x) = \frac{1}{x^4}$$

per $x \longrightarrow -\infty$

Se $f(x) \underset{x \to x_0}{\longrightarrow} +\infty$ e $g(x) \underset{x \to x_0}{\longrightarrow} +\infty$

$[\, x_0 \in \overline{\mathbb{R}} \,]$ e se $\lim\limits_{x \to x_0} \frac{f(x)}{g(x)} = +\infty$

f è un INFINITO
di ORDINE SUPERIORE
rispetto a g per $x \longrightarrow x_0$.

IN PARTICOLARE $\lim\limits_{x \to x_0} \frac{g(x)}{f(x)} = 0$

e si dice anche CHE
g è un INFINITO di ORDINE
INFERIORE
rispetto a f

$$g(x) = o(f(x)), \quad x \to x_0$$

In particolare g è molto più piccolo di f definitivamente per $x \to x_0$

Es. $f(x) = \frac{1}{x^4}$, $g(x) = \frac{1}{x^2}$

$x \to 0$

Attenzione

$$f(x) = \frac{1}{x^5} \xrightarrow[x \to 0^\pm]{} \pm\infty$$

In questi casi si confrontano i moduli delle funzioni.

$f(x) = \frac{1}{|x|^5}$, $\quad g(x) = \frac{1}{x^2}$

per $x \to 0$

$$f(x) = \frac{1}{|x-1|^3}, \quad g(x) = \frac{1}{|x-1|^2}$$

per $x \to 1$

$$f(x) = e^{\frac{1}{|x-1|}}, \quad g(x) = \frac{1}{|x-1|^5}$$

per $x \to 1$

$$f(x) = \frac{1}{|x-1|^{\frac{1}{3}}}, \quad g(x) = \left|\log|x-1|\right|$$

VEDIAMO PERCHE' SE

$$f \xrightarrow[x \to x_0]{} 0, \quad g \xrightarrow[x \to x_0]{} 0 \quad e$$

$$\frac{f(x)}{g(x)} \to 0, \quad x \to x_0$$

ALLORA f è MOLTO PIÙ PICCOLO di g $(f << g)$

DEFINITIVAMENTE per $x \to x_0$

PER semplicità supponiamo
$\mathrm{dom}(f) = \mathbb{R} \setminus \{x_0\} = \mathrm{dom}(g)$, $x_0 \in \mathbb{R}$
e $g(x) \neq 0$ definitivamente per $x \to x_0$

$$\forall \varepsilon > 0 \ \exists \delta_\varepsilon > 0 : \left| \frac{f(x)}{g(x)} - 0 \right| < \varepsilon$$
$$\forall x \in (x_0 - \delta_\varepsilon, x_0 + \delta_\varepsilon) \setminus \{x_0\}$$

$$\frac{|f(x)|}{|g(x)|} < \varepsilon \Rightarrow |f(x)| < \varepsilon |g(x)|$$
($g(x) \neq 0$ definitivamente)

Quindi $\forall \varepsilon > 0$ si ha
$$|f(x)| < \varepsilon |g(x)|$$
definitivamente per $x \to x_0$.

Nel caso $f(x) \xrightarrow[x \to x_0]{} +\infty$, $g(x) \xrightarrow[x \to x_0]{} +\infty$

e $\frac{f(x)}{g(x)} \longrightarrow +\infty$, $x \longrightarrow x_0$.

$\forall M > 0 \ \exists \delta_M > 0 : \ \frac{f(x)}{g(x)} > M$

$\forall x \in (x_0 - \delta_M, x_0 + \delta_M) \setminus \{x_0\}$

$\forall M > 0 \quad f(x) > M g(x)$

definitivamente per $x \to x_0$