FORMULA del BINOMIO di NEWTON

$$(a+b)^m = \sum_{k=0}^{m} \binom{m}{k} a^{m-k} b^k$$

$a \in \mathbb{R}$, $b \in \mathbb{R}$, $m \in \mathbb{N}$

$$\binom{m}{k} = \frac{m!}{k!\,(m-k)!}$$

$$(a+b)^1 = \binom{1}{0} a^{1-0} b^0 + \binom{1}{1} a^{1-1} b^1 = a+b$$

$$\boxed{0! = 1}$$

$$\binom{1}{0} = \frac{1!}{0!\,(1-0)!} = \frac{1}{0! \cdot 1} = 1$$

$$\binom{1}{1} = \frac{1!}{1!\,0!} = \frac{1}{1 \cdot 0!} = 1$$

$$(a+b)^2 = \binom{2}{0} a^2 + \binom{2}{1} ab + \binom{2}{2} b^2$$

$$\binom{2}{0} = \frac{2!}{0!(2-0)!} = \frac{2!}{0!\,2!} = 1 = \binom{2}{2} = \frac{2!}{0!(2-0)!}$$

$$\boxed{\binom{m}{0} = 1 = \binom{m}{m}}$$

$$= a^2 + 2ab + b^2 .$$

$$(a+b)^3 = \binom{3}{0} a^3 + \binom{3}{1} a^2 b + \binom{3}{2} a b^2 + \binom{3}{3} b^3$$

$$= a^3 + 3a^2 b + 3ab^2 + b^3$$

$$\binom{3}{1} = \frac{3!}{1!(3-1)!} = \frac{3!}{1\cdot 2!} = \frac{3\cdot 2\cdot 1}{2\cdot 1} = 3$$

$$\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3!}{2!\,1!} = 3$$

$$\boxed{\binom{m}{1} = m = \binom{m}{m-1}}$$

TRIANGOLO di TARTAGLIA

$m = 0$ 1

$m = 1$ $\binom{1}{0}\ 1 \quad 1\ \binom{1}{1}$

$m = 2$ $\binom{2}{0}\ 1 \quad 2 \quad 1\ \binom{2}{2}$

$m = 3$ $\binom{3}{0}\ 1 \quad 3 \quad 3 \quad 1\ \binom{3}{3}$

$m = 4$ $\binom{4}{0}\ 1 \quad 4 \quad 6 \quad 4 \quad 1\ \binom{4}{4}$

$m = 5$ $\binom{5}{0}\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1\ \binom{5}{5}$

$$\binom{5}{1} \binom{5}{2} \binom{5}{3} \binom{5}{4}$$

$$10 = \binom{5}{2} = \binom{4}{1} + \binom{4}{2} = 4 + 6$$

$$\boxed{\binom{m+1}{k} = \binom{m}{k-1} + \binom{m}{k}}$$

$$\boxed{\binom{m}{k} = \binom{m}{m-k}}$$

DIM.

$$\frac{m!}{k!\,(m-k)!} = \frac{m!}{(m-k)!\,k!} = \frac{m!}{(m-k)!\,(m-(m-k))!}$$

$a = 1$, $b = x$

$$(1+x)^m = \sum_{k=0}^{m} \binom{m}{k} x^k =$$

$$= \underset{\binom{n}{0}}{1} + \underset{\binom{n}{1}}{nx} + \underset{\binom{n}{2}}{\frac{n(n-1)}{2}} x^2 + \dots + \underset{\binom{n}{n-1}}{n\, x^{n-1}} + \underset{\binom{n}{n}}{x^n}$$

$a > 1$, $\boxed{a^n} = (a - 1 + 1)^n$

$$= (1 + (a-1))^n = \sum_{k=0}^{n} \binom{n}{k} (a-1)^k$$

$a - 1 \leftrightarrow x$

$$= 1 + n(a-1) + \frac{n(n-1)}{2}(a-1)^2 + \dots + (a-1)^n$$

$$\boxed{>} \quad \boxed{\frac{n(n-1)}{2}(a-1)^2}, \quad n \geq 2 .$$

$$(1+1)^n = 2^n = \sum_{k=0}^{n} \binom{n}{k} = \boxed{\text{\# di sottoinsiemi di un insieme di } n \text{ elementi}}$$

$\binom{n}{k}$ = combinazioni di n elementi di classe k =

= # di sottoinsiemi di k elementi in un insieme di n elementi

$$\binom{3}{2} \quad \{1,2\}, \{1,3\}, \{2,1\}, \{2,3\}, \{3,1\}, \{3,2\}$$

$$\{1,2\}, \{1,3\}, \{2,3\}$$

$$\binom{m+1}{k} = \frac{(m+1)!}{k!\,(m+1-k)!}$$

DIMOSTRAZIONE

$$\binom{m+1}{k} = \binom{m}{k-1} + \binom{m}{k} =$$

$$= \frac{m!}{(k-1)!\,(m-k+1)!} + \frac{m!}{k!\,(m-k)!} =$$

$$= \frac{m!\,k!\,(m-k)! + m!\,(k-1)!\,(m-k+1)!}{(k-1)!\,(m-k+1)!\,k!\,(m-k)!}$$

$$k! = k\,((k-1)!)$$

$$(m-k+1)! = (m-k+1)\cdot(m-k)!$$

$$= n!\,\frac{k\cdot(k-1)!\,(n-k)! + (k-1)!\,(n-k+1)\cdot(n-k)!}{(k-1)!\,(n-k+1)!\,k!\,(n-k)!}$$

$$= n!\,\cancel{(k-1)!}\,\cancel{(n-k)!}\;\frac{\cancel{k} + n - \cancel{k} + 1}{\cancel{(k-1)!}\,(n-k+1)!\,k!\,\cancel{(n-k)!}}$$

$$= \frac{n!\,(n+1)}{k!\,(n-k+1)!} = \frac{(n+1)!}{k!\,(n-k+1)!} = \binom{n+1}{k} \quad \blacksquare$$

DIMOSTRAZIONE della FORMULA del BINOMIO

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}\, b^k$$

$\underline{a \neq 0}$ $\quad a^n \left(1 + \frac{b}{a}\right)^n = (a+b)^n$

$$x = \frac{b}{a} \qquad \boxed{(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k}$$

PER INDUZIONE $\quad P_n$

$$n=0\ ;\quad 1=(1+x)^0=\sum_{k=0}^{0}\binom{0}{0}x^0=\binom{0}{0}\cdot 1$$

$$\frac{0!}{0!\,(0-0)!}=\frac{0!}{0!\,0!}=1$$

$$n=1\ ;\quad (1+x)^1=\sum_{k=0}^{1}\binom{1}{k}x^k=\binom{1}{0}x^0+\binom{1}{1}x^1$$

$$=1+x$$

(i) $\mathcal{P}_1$ E' VERA

(ii) $n\in\mathbb{N}$, $(1+x)^n=\sum_{k=0}^{n}\binom{n}{k}x^k\implies$

$$(1+x)^{n+1}=\sum_{k=0}^{n+1}\binom{n+1}{k}x^k$$

$$(1+x)(1+x)^n=(1+x)\sum_{k=0}^{n}\binom{n}{k}x^k=$$

$$=\sum_{k=0}^{n}\binom{n}{k}x^k+x\sum_{k=0}^{n}\binom{n}{k}x^k$$

$$\boxed{x \sum_{k=0}^{m} \binom{m}{k} x^k = \sum_{k=0}^{m} \binom{m}{k} x^{k+1}}$$

$$x\,(1 + m x + \dots) = x + m x^2 + \dots$$

$m = k+1$, $k = m - 1$

$$\sum_{k=0}^{m} \binom{m}{k} x^{k+1} = x^1 + m x^2 + \dots + x^{m+1} =$$

$$= \sum_{m=1}^{m+1} \binom{m}{m-1} x^m$$

$$\sum_{k=0}^{m} \binom{m}{k} x^{k+1} = \sum_{m=1}^{m+1} \binom{m}{m-1} x^m$$

$m = k+1$, $k = m - 1$

CAMBIAMENTO di VARIABILE NELLA SOMMA

$m = k+1$

$k = 0 \Longrightarrow m = 1$

$k = m \Longrightarrow m = m+1$

$$\sum_{k=0}^{m} \binom{m}{k} x^{k+1} = \sum_{k=1}^{m+1} \binom{m}{k-1} x^{k}$$

$$(1+x)^{m+1} = \sum_{k=0}^{m} \binom{m}{k} x^{k} + \sum_{k=0}^{m} \binom{m}{k} x^{k+1}$$

$$= \sum_{k=0}^{m} \binom{m}{k} x^{k} + \sum_{k=1}^{m+1} \binom{m}{k-1} x^{k} =$$

$$= \binom{m}{0} + \sum_{k=1}^{m} \binom{m}{k} x^{k} + \sum_{k=1}^{m} \binom{m}{k-1} x^{k} + \binom{m}{m+1-1} x^{m+1}$$

$$= 1 + \sum_{k=1}^{m} \left(\binom{m}{k} + \binom{m}{k-1} \right) x^{k} + x^{m+1} =$$

$$\binom{m+1}{k} = \binom{m}{k-1} + \binom{m}{k}$$

$$= 1 + \sum_{k=1}^{m} \binom{m+1}{k} x^{k} + x^{m+1} =$$

$$= \binom{n+1}{0} + \sum_{k=1}^{n} \binom{n+1}{k} x^k + \binom{n+1}{n+1} x^{n+1}$$

$$= \sum_{k=0}^{n+1} \binom{n+1}{k} x^k \qquad \blacksquare$$

$$\sigma_m(n) = \sum_{k=1}^{n} k^m \, ,$$

$$\sigma_1(n) = \sum_{k=1}^{n} k \, ,$$

$$\sigma_2(n) = \sum_{k=1}^{n} k^2 \, ,$$

$$\sigma_3(n) = \sum_{k=1}^{n} k^3 \, ,$$

$$\sum_{k=1}^{m} \binom{m}{k} \sigma_{m-k}(n) = (n+1)^m - 1 \qquad (*)$$

$$m = 1 \, , \quad \binom{1}{1} \sigma_0(n) = n$$

$$\sigma_0(n) = n \, .$$

$$\sigma_0(m) = \underbrace{1+1+1\dots+1}_{m \text{ volte}} = m$$

$$m=2, \quad \sum_{k=1}^{2} \binom{2}{k} \sigma_{2-k}(m) = (m+1)^2 - 1$$

$$\binom{2}{1}\sigma_1(m) + \binom{2}{2}\sigma_0(m) = m^2 + 2m$$

$$2\sigma_1(m) + \sigma_0(m) = m^2 + 2m$$

$$2\sigma_1(m) + m = m^2 + 2m$$

$$2\sigma_1(m) = m^2 + m \Rightarrow \sigma_1(m) = \frac{m(m+1)}{2}$$

$$\sigma_2(m) = \frac{m(m+1)(2m+1)}{6}$$

PER INDUZIONE

✱ $m=3$

$$\sigma_3(m) = \frac{m^2(m+1)^2}{4}$$

✱ $m=4$

$$\sqrt{x^2-2x} > 1-2x \quad (1)$$

$$\begin{cases} x^2-2x \geq 0 \\ 1-2x \leq 0 \end{cases} \cup \begin{cases} \cancel{x^2-2x \geq 0} \\ 1-2x > 0 \\ x^2-2x > (1-2x)^2 \end{cases}$$

$$\begin{cases} x < 0 \vee x > 2 \\ x \geq \frac{1}{2} \end{cases} \Rightarrow \{(-\infty,0) \cup (2,+\infty)\} \cap \left[\frac{1}{2},+\infty\right) = (2,+\infty)$$

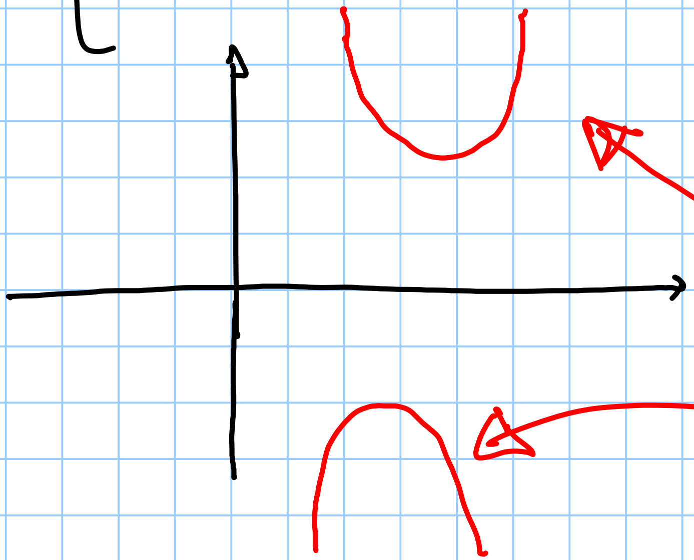
(PARENTESI GRAFFE) NECESSARIE!!!

N.B. $A \cup (B \cap C) \neq (A \cup B) \cap C =$
$$= (A \cap C) \cup (B \cap C)$$

$$\begin{cases} 1-2x > 0 \\ x^2-2x > (1-2x)^2 \end{cases} \quad \begin{cases} x < \frac{1}{2} \\ x^2-2x > 1-4x+4x^2 \end{cases}$$

$$\begin{cases} x < \frac{1}{2} \\ 3x^2 - 2x + 1 < 0 \end{cases} \longrightarrow \text{MAI}$$

$\Delta = 4 - 12 < 0$



$f(x) = ax^2 + bx + c$

$(a > 0 \quad \Delta < 0)$

$(a < 0 \quad \Delta < 0)$

SOLUZIONE di (1)

$$(2, +\infty) \cup \emptyset = (2, +\infty)$$

N.B. $x^2 - 2x > (1 - 2x)^2$ non è MAI VERIFICATA, (1) è VERIFICATA in $(2, +\infty)$

$$\sqrt{2x^2+6x+11} > 3 - \frac{x}{2}$$

$$\begin{cases} 2x^2+6x+11 \geq 0 \\ 3 - \frac{x}{2} < 0 \end{cases} \quad \cup \quad \begin{cases} 3 - \frac{x}{2} \geq 0 \\ \cancel{2x^2+6x+11} \geq 0 \\ 2x^2+6x+11 > \left(3 - \frac{x}{2}\right)^2 \end{cases}$$

$\to$ E' VERA $\forall x$ CHE VERIFICA

$$\begin{cases} 2x^2+6x+11 \geq 0 \\ 3 - \frac{x}{2} < 0 \end{cases} \cup \begin{cases} 2x^2+6x+11 > \left(3 - \frac{x}{2}\right)^2 \\ 3 - \frac{x}{2} \geq 0 \end{cases}$$

$$\sqrt{2x^2+6x+11} < 3 - \frac{x}{2}$$

$$\begin{cases} 2x^2+6x+11 \geq 0 \\ 3 - \frac{x}{2} > 0 \\ 2x^2+6x+11 < \left(3 - \frac{x}{2}\right)^2 \end{cases}$$

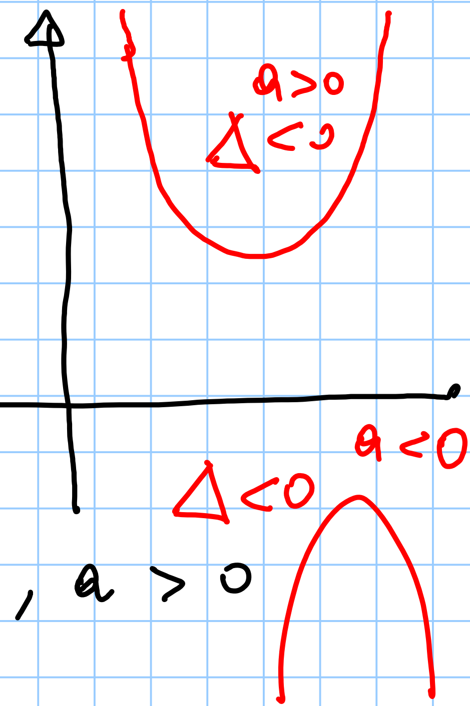
Ex

$$\begin{cases} 2x^2+6x+11 \geq 0 \\ 3-\frac{x}{2} < 0 \\ x > 6 \end{cases} \cup \begin{cases} 2x^2+6x+11 > \left(3-\frac{x}{2}\right)^2 \\ 3-\frac{x}{2} \geq 0 \\ x \leq 6 \end{cases}$$

$$2x^2+6x+11 \geq 0$$

$$x_{\pm} = -3 \pm \sqrt{9-22} \qquad \Delta < 0$$

$$\Delta = b^2-4ac<0$$



$$f(x) = ax^2+bx+c$$

$$\Delta = b^2-4ac<0 \quad , \; a>0$$

$$\boxed{2x^2+6x+11>0 \quad \forall x \in \mathbb{R}}$$

$$\mathbb{R} \cap \{x>6\}$$

$$= \{x>6\}$$

$$2x^2+6x+11>\left(3-\frac{x}{2}\right)^2,\quad x\le 6$$

$$2x^2+6x+11>9-3x+\frac{x^2}{4},$$

$$\frac{7}{4}x^2+9x+2>0$$

$$x_\pm=\frac{-9\pm\sqrt{81-14}}{7/2}=$$

$$=\left(-9\pm\sqrt{67}\right)\cdot\frac{2}{7}$$

$$\sqrt{64}<\sqrt{67}<\sqrt{81};\ 8<\sqrt{67}<9$$

$$-1<\sqrt{67}-9<0$$

$$\left\{x<-\frac{2}{7}\left(9+\sqrt{67}\right)\right\}\cup$$

$$\left\{\underset{\overset{\|}{x_+}}{\left(-9+\sqrt{67}\right)\frac{2}{7}}<x\le 6\right\}$$

$$x_+<0<6$$

$$\sqrt{2x^2+6x+11} > 3-\frac{x}{2} \iff$$

$$x \in (-\infty, x_-) \cup (x_+, +\infty)$$

$$x_- = -\frac{2}{7}\left(9+\sqrt{67}\right)$$

$$x_+ = \frac{2}{7}\left(-9+\sqrt{67}\right)$$

$$\sqrt{2x^2+6x+11} < 3-\frac{x}{2} \iff$$

$$x \in (x_-, x_+)$$