$$f(x)=\frac{1}{x^2-x}$$

$$\mathrm{dom}(f)=\{x\in\mathbb{R}:\ x^2-x\neq 0\}$$

$$x^2-x=0 \iff x(x-1)=0$$

$x=0$ o $x=1$

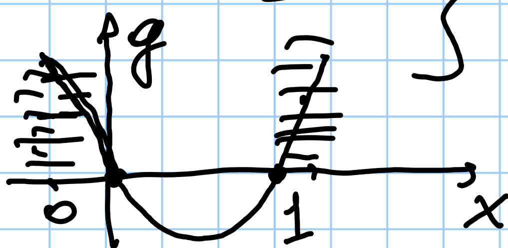
$$\mathrm{dom}(f)=\mathbb{R}\setminus\{0,1\}=$$

$$=(-\infty,0)\cup(0,1)\cup(1,+\infty)$$

$$f(x)=\sqrt{x^2-x}$$

$$\mathrm{dom}(f)=\{x\in\mathbb{R}:\ x^2-x\geq 0\}$$

$$g(x)=x^2-x$$



$$\mathrm{dom}(f) = (-\infty, 0] \cup [1, +\infty) = \mathbb{R} \setminus (0,1)$$

$$f(x) = \log_2(x^2 - x)$$

$$\mathrm{dom}(f) = \{ x \in \mathbb{R} : x^2 - x > 0 \} = (-\infty, 0) \cup (1, +\infty) = \mathbb{R} \setminus [0,1]$$

$$f(x) = \log_{10}\left(\log_2(x^2 - x)\right)$$

$$\mathrm{dom}(f) = \{ x \in \mathbb{R} : x^2 - x > 0 \text{ e } \log_2(x^2-x) > 0 \}$$

$$= \underbrace{\{ x \in \mathbb{R} : x^2 - x > 0 \}}_{B} \cap \underbrace{\{ x \in \mathbb{R} : \log_2(x^2 - x) > 0 \}}_{A}$$

$$(x^2 - x > 1)$$

$$x \in A \Downarrow x \in B$$

$$\log_2(x^2-x)>0$$

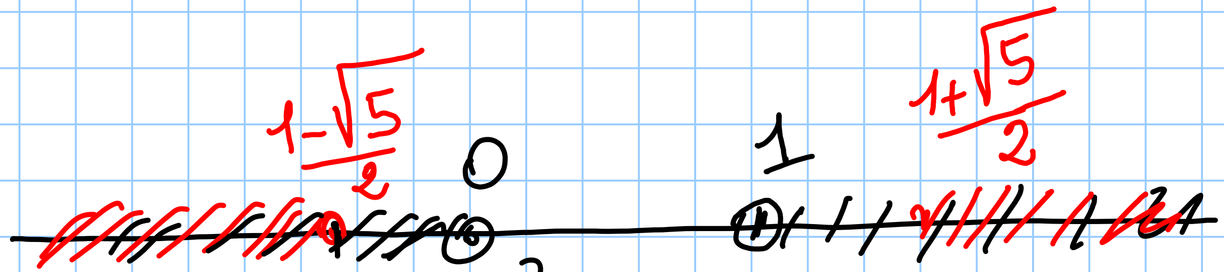
$$2^{\log_2(x^2-x)} > 2^0$$

$$x^2-x>1$$

$$x^2-x-1>0 \qquad x_\pm = \frac{1\pm\sqrt{5}}{2}$$

$$x\in\left(-\infty,\frac{1-\sqrt{5}}{2}\right)\cup\left(\frac{1+\sqrt{5}}{2},+\infty\right)$$

$$\mathrm{dom}(f)=\left(-\infty,\frac{1-\sqrt{5}}{2}\right)\cup\left(\frac{1+\sqrt{5}}{2},+\infty\right)=$$



$$=\underbrace{\{(-\infty,0)\cup(1,+\infty)\}}_{B}\cap\underbrace{\left\{\left(-\infty,\frac{1-\sqrt{5}}{2}\right)\cup\left(\frac{1+\sqrt{5}}{2},+\infty\right)\right\}}_{A}$$

$\Downarrow$

$A\subseteq B$

$\Downarrow$

$A\cap B=A$

$$\mathrm{dom}(f) = \{x \in \mathbb{R} : x^2 - x > 0\} \cap \{x \in \mathbb{R} : x^2 - x > 1\} = \{x \in \mathbb{R} : x^2 - x > 1\}$$

DEFINIZIONE

$I \subseteq \mathbb{R}$ SI DICE SIMMETRICO

SE $x \in I \Rightarrow -x \in I$

$I = (-a, a)$ $\boxed{\text{SONO SIMMETRICI}}$

$I = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$

DEFINIZIONE

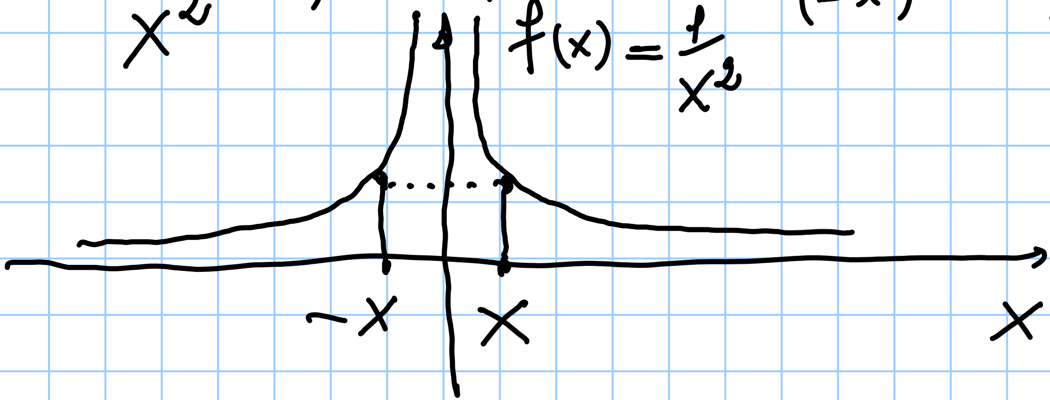
Se $I \subseteq \mathbb{R}$ è SIMMETRICO e

$f : I \to \mathbb{R}$, ALLORA

SI DICE CHE f è PARI se $f(x)=f(-x)$,
SI DICE CHE f è DISPARI se $\boxed{\forall x \in I}$
$f(x)=-f(-x)$.

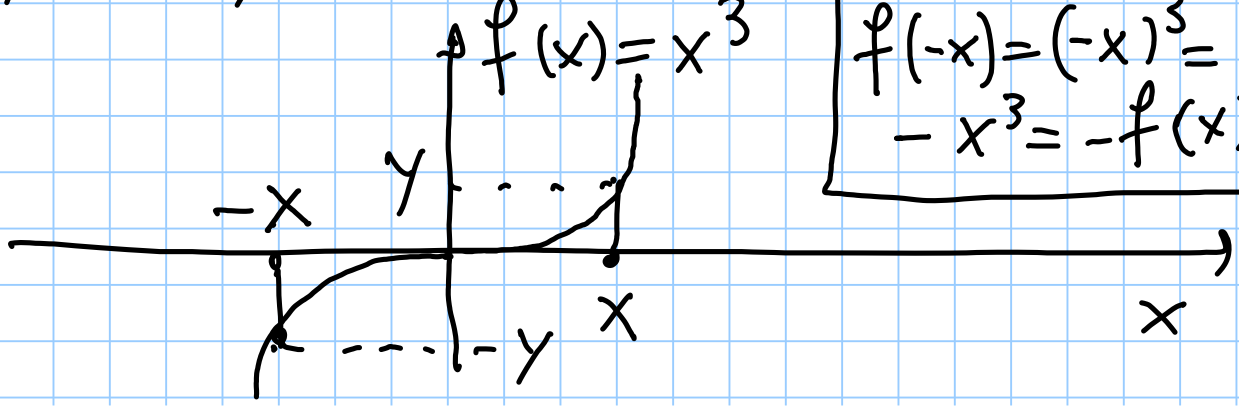
$f(x)=x^2$, $\frac{1}{x^2}$ SONO PARI
nei RISPETTIVI DOMINI NATURALI

$f(x)=\frac{1}{x^2}$, $f(-x)=\frac{1}{(-x)^2}=\frac{1}{x^2}=f(x)$



$f(x)=x$, x^3 SONO DISPARI
in $\mathbb{R}$

$f(-x)=(-x)^3=$
$-x^3=-f(x)$



DEFINIZIONE $X \subseteq \mathbb{R}$

$f : X \longrightarrow \mathbb{R}$. f è

SUPERIORMENTE LIMITATA

SE $\mathrm{Im}(f)$ è SUPERIORMENTE LIMITATO , OVVERO

$\exists M \in \mathbb{R} : f(x) \le M, \forall x \in X$.

Si dice ESTREMO SUPERIORE di f in X , e si indica con

$\left(\inf_X f\right)$ $\sup_X f := \sup \{\mathrm{Im}(f)\}$

si dice MASSIMO di f in X

$\left(\min_X f\right)$ $\max_X f := \max \{\mathrm{Im}(f)\}$

SE $\exists$ IL MASSIMO

DEFINIZIONE $X \subseteq \mathbb{R}$

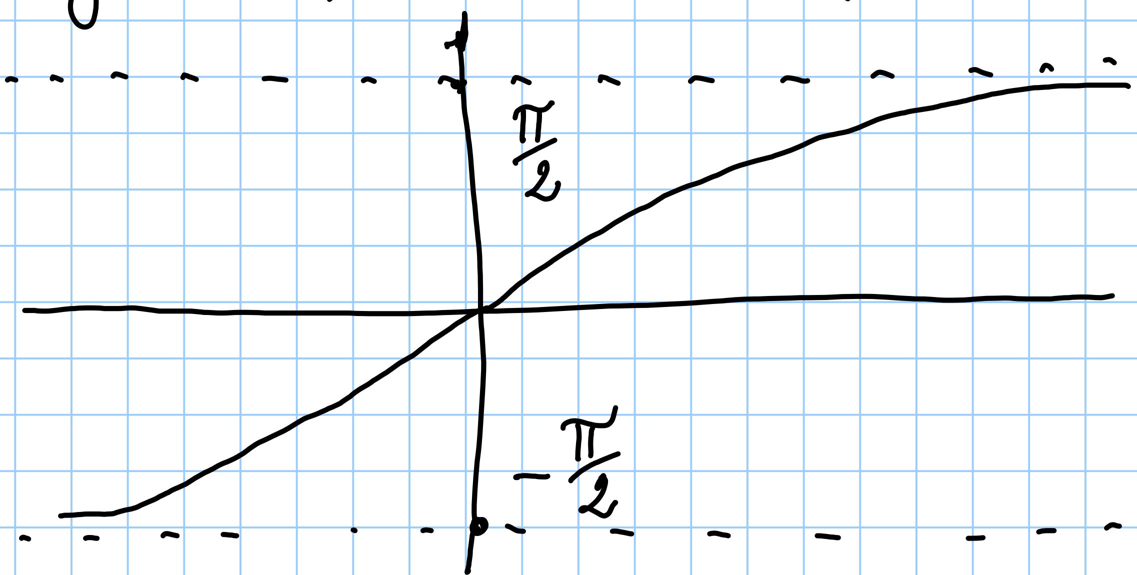
$f : X \to \mathbb{R}$. f si dice

LIMITATA se è

superiormente e inferiormente limitata.

ILLIMITATA SE NON È LIMITATA.

$f(x) = \operatorname{arcTg}(x)$, $\operatorname{dom}(f) = (-\infty, +\infty)$



f è LIMITATA, $\operatorname{Im}(f) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\inf_{\mathbb{R}} f = -\frac{\pi}{2}$, $\sup_{\mathbb{R}} f = \frac{\pi}{2}$

$f(x) = \cos(x) \quad , x \in \mathbb{R}$

$\mathrm{Im}(f) = [-1, 1]$

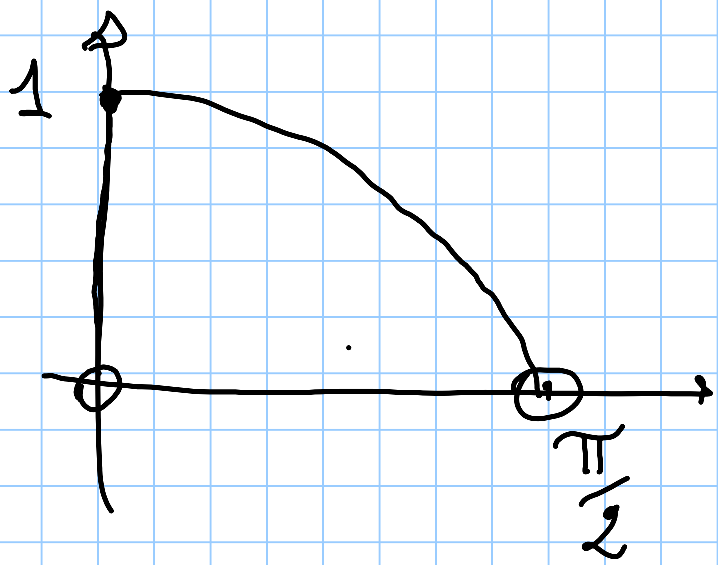
$-1 = \min_{\mathbb{R}} f = f(\pi) = f(3\pi) = \dots$

$1 = \max_{\mathbb{R}} f = f(0) = f(2\pi) = \dots$

$f(x) = \cos(x), \quad x \in X$

$X = [0, \frac{\pi}{2})$

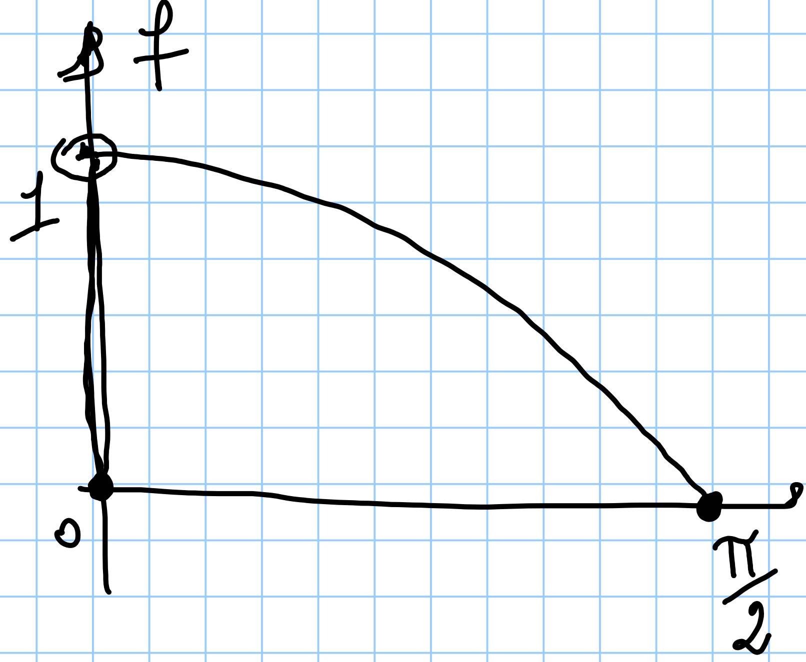
$\mathrm{Im}(f) = (0, 1]$



$\max_{[0, \frac{\pi}{2})} f = 1$

$\inf_{[0, \frac{\pi}{2})} f = 0$

$$X = \left(0, \frac{\pi}{2}\right]$$



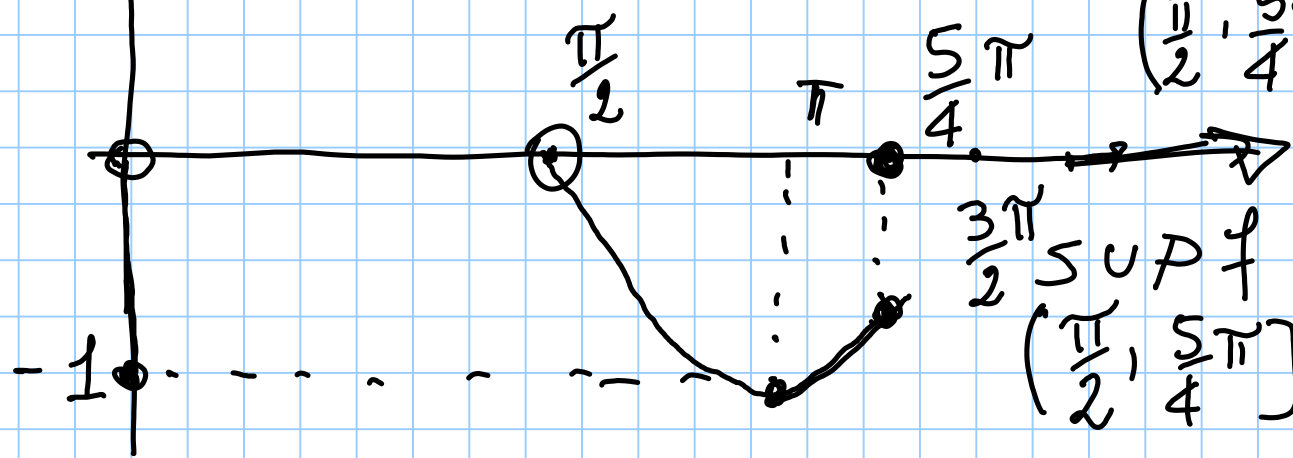
$$\operatorname{Im}(f) = [0, 1)$$

$$\min_{\left(0, \frac{\pi}{2}\right]} f = 0$$

$$\sup_{\left(0, \frac{\pi}{2}\right]} f = 1$$

$$X = \left(\frac{\pi}{2}, \frac{5}{4}\pi\right]$$

$$\operatorname{Im}(f) = [-1, 0)$$



$$\min_{\left(\frac{\pi}{2}, \frac{5\pi}{4}\right]} f = -1$$

$$\sup_{\left(\frac{\pi}{2}, \frac{5\pi}{4}\right]} f = 0$$

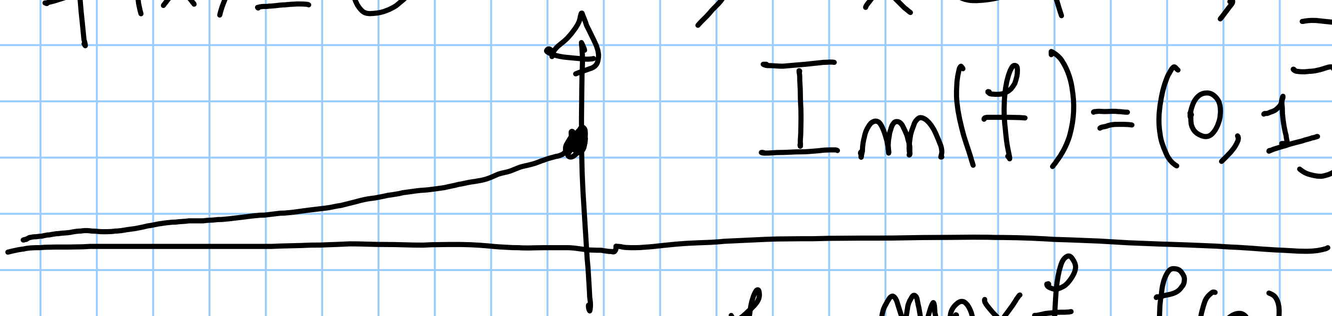
$f(x) = e^x$, $x \in \mathbb{R}$

f è ILLIMITATA,

$\sup_{\mathbb{R}} f = +\infty$,

$\inf_{\mathbb{R}} f = 0$ e inferiormente LIMITATA

$f(x) = e^x$, $x \in (-\infty, 0]$



$Im(f) = (0, 1]$

$1 = \max_{(-\infty,0]} f = f(0)$

$\inf_{(-\infty,0]} f = 0$

DEFINIZIONE $X \subseteq \mathbb{R}$.

$f : X \to \mathbb{R}$.

f si dica MONOTONA

CRESCENTE

(STRETTAMENTE CRESCENTE)

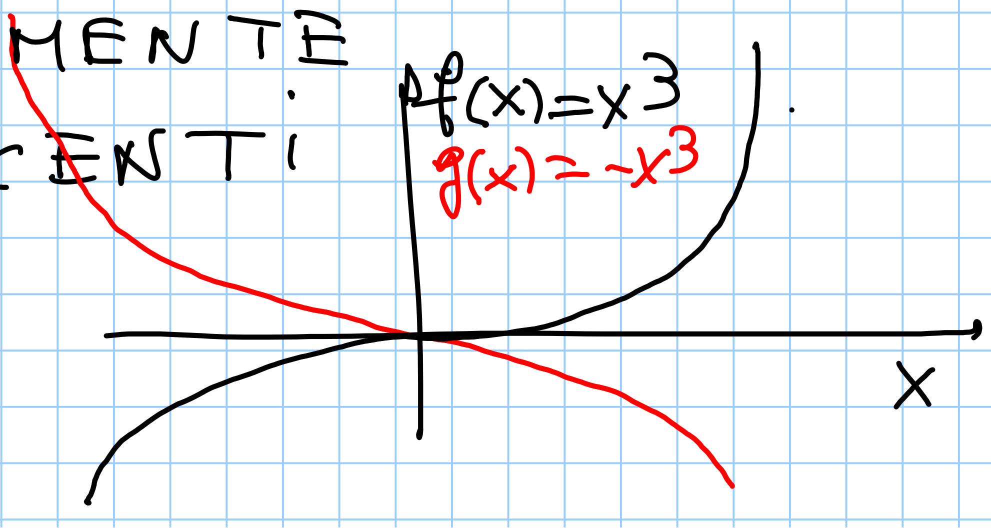
$$x < y \Rightarrow f(x) \leq f(y)$$
$$(f(x) < f(y))$$

$f(x) = x,\ x^3,\ 3^x,\ \log_3 x, \dots$

MONOTONE STRETTAMENTE CRESCENTI nei RISPETTIVI DOMINI NATURALI

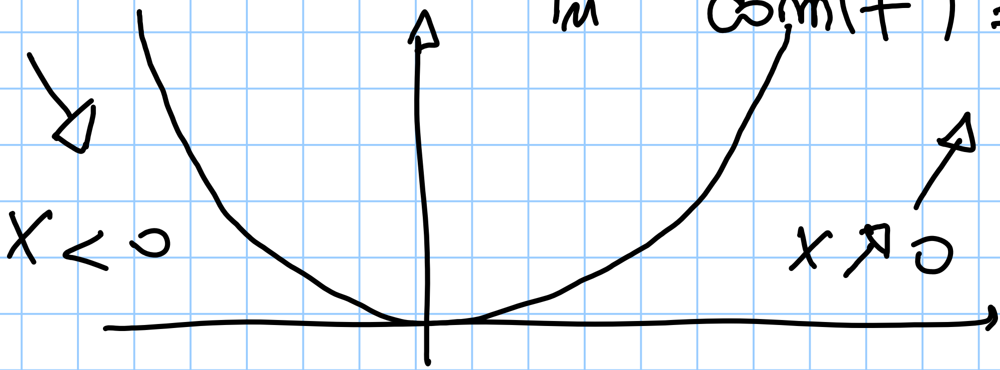
$f(x) = -x,\ -x^3,\ -3^x,\ -\log_3 x, \dots$

STRETTAMENTE DECRESCENTI



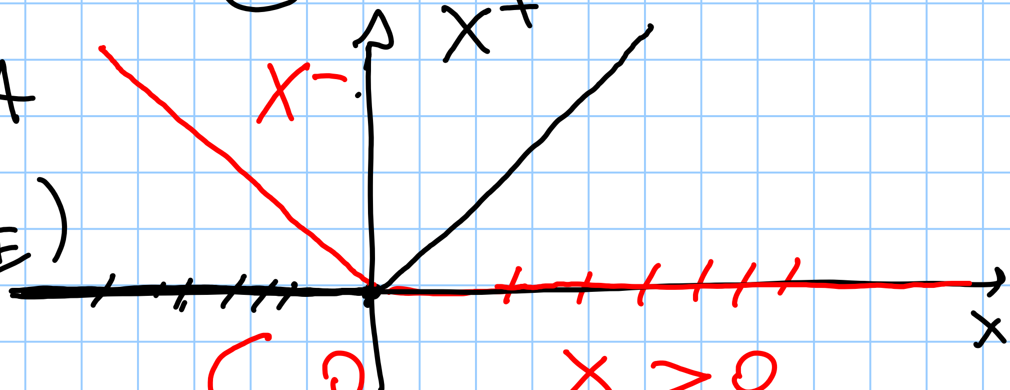
$f(x) = x^2$

NON E' MONOTONA in $dom(f) = \mathbb{R}$



$$f(x) = x^+ = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

PARTE POSITIVA (MONOTONA CRESCENTE)



$$g(x) = x^- = \begin{cases} 0, & x > 0 \\ -x, & x \leq 0 \end{cases}$$

PARTE NEGATIVA (MONOTONA DECRESCENTE)

OSSERVAZIONE

$|x| = x^+ + x^-$

TEOREMA

(i) Sia $f: [a,b] \longrightarrow \mathbb{R}$, CRESCENTE.

ALLORA $\min_{[a,b]} f = f(a)$, $\max_{[a,b]} f = f(b)$

(ii) Sia $f: (a,b) \longrightarrow \mathbb{R}$, STRETTAMENTE CRESCENTE.

ALLORA $\nexists \min_{(a,b)} f$, $\nexists \max_{(a,b)} f$

DIMOSTRAZIONE

(i) $f(a) \leq f(x) \quad \forall x \in [a,b]$

f è CRESCENTE.

$f(a)$ è MINORANTE di $Im(f)$.

$f(a) \in Im(f) \Longrightarrow f(a) = \min_{[a,b]} f$

(ii) PER ASSURDO

SUPPONIAMO CHE $\exists \min_{(a,b)} f$

$$\exists\ x_m \in (a,b) : f(x_m) \le f(y)$$
$$a < x_m < b \qquad \forall y \in (a,b)$$

Ma f è STRETTAMENTE CRESCENTE $\Longrightarrow$

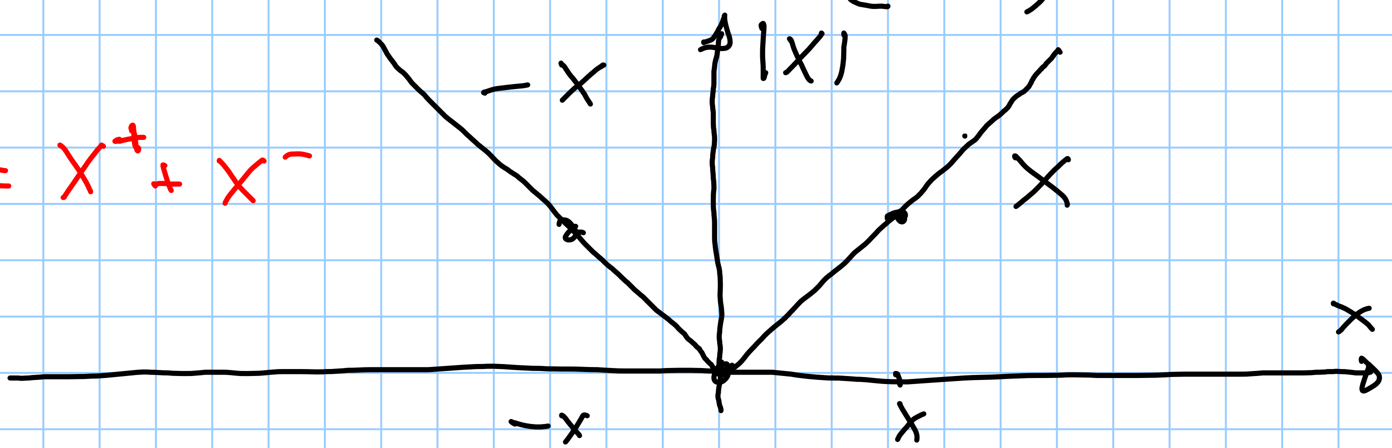
$$y \in (a, x_m) \Longrightarrow y < x_m$$
$$f(y) < f(x_m) \ (\le f(y))$$
$$f(y) < f(y)$$ ASSURDO

ESERCIZIO: COMPLETARE LA DIMOSTRAZIONE ■

FUNZIONE MODULO

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = |x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$

$$|x| = x^+ + x^-$$



(i) $|x| \geq 0$, $|x| = 0 \iff x = 0$

(ii) $|x+y| \leq |x| + |y|$, ⊛

$||x| - |y|| \leq |x-y|$

(iii) $|x \cdot y| = |x| \cdot |y|$

(ii) $x = 0$ o $y = 0$ DiM.

$|x+y| = |x| = |x| + |y|$

VALE = in ⊛

$x \neq 0$ e $y \neq 0$

• $\boxed{x > 0 \text{ e } y > 0}$

$|x+y| = x+y = |x| + |y|$, VALE = in ⊛

• $x < 0$, $y > 0$

$|x+y| = |-|x| + y| = \max\{-|x| + y, |x| - y\} <$

ESEMPIO: $x = -5$, $y = 3$. $-|x| + y = -5 + 3 = -2$

$|x| - y = 5 - 3 = 2$

$< \quad |x| + y = |x| + |y| \quad$ VALE $<$ in $\circledast$

$x_1 = y < 0$
$y_1 = x > 0$

- $x > 0 \;,\; y < 0$

$$|x+y| = |y_1 + x_1| = |x_1 + y_1| < |x_1| + |y_1| =$$
$$= |y| + |x| = |x| + |y| .$$

- $x < 0 \,,\; y < 0$

$$|x+y| = \big| -|x| - |y| \big| = \big| -(|x| + |y|) \big|$$
$$= |x| + |y|$$

$\circledast$ VALE $=$ in $\circledast$

$$\big| |x| - |y| \big| \leq |x - y| \quad \underline{\text{DiM.}}$$

$$|x| = |\underbrace{x-y}_{x_1} + \underbrace{y}_{y_1}| = |x_1 + y_1| \leq$$
$$\leq |x_1| + |y_1| = |x-y| + |y|$$
$$|x| \leq |x-y| + |y|$$

$|x| - |y| \leq |x-y|$

$$|y| = |y-x+x| \leq \dots \leq |y-x| + |x|$$

$|y| - |x| \leq |y-x| = |x-y|$

$$-|x-y| \leq |x|-|y| \leq |x-y| \iff$$

$$\boxed{\big||x|-|y|\big| \leq |x-y|}$$

$\blacksquare$

IN PARTICOLARE ABBIAMO DIMOSTRATO CHE:

$|x+y| \leq |x| + |y|$ e CHE VALE $<$ SE E SOLO SE $x \neq 0$ e $y \neq 0$ e $x \cdot y < 0$