DEFINIZIONE

RETTA REALE ESTESA

$$\overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty, +\infty\}$$

LA RELAZIONE d'ORDINE ($\leq$) VERIFICA (i) - (ii) - (iii)

e

(iv) $-\infty \leq x \leq +\infty \quad \forall x \in \overline{\mathbb{R}}$

$$A = \{x \in \mathbb{R} : x < \pi\}$$

$$\pi = \sup A$$

$$\inf A = -\infty$$

SE UN INSIEME A NON HA MAGGIORANTI

$\Longrightarrow \quad \sup A := +\infty$

SE NON HA MINORANTI

$\Longrightarrow \quad \inf A := -\infty$

$+\infty$ e $-\infty$ NON SONO NUMERI REALI

$x \in \mathbb{R}$

(a) $+\infty \pm x = +\infty$;

$-\infty \pm x = -\infty$; $\quad \dfrac{x}{\pm\infty} = 0$.

(b) SE $x > 0$ ALLORA $\pm\infty \cdot x = \pm\infty$

SE $x < 0$ ALLORA $\pm\infty \cdot x = \mp\infty$

(c) $(+\infty) \cdot (\pm\infty) = \pm\infty$,

$$(\pm\infty)+(\pm\infty)=\pm\infty$$

FORME INDETERMINATE

(i) $+\infty \pm (\mp\infty)$

(ii) $\dfrac{\pm\infty}{\pm\infty}$, $\dfrac{\pm\infty}{\mp\infty}$

(iii) $\pm\infty \cdot 0$

$+\infty-(+\infty)=0$?

$5+(+\infty)-(+\infty)=5$

LA SOMMA DEVE ESSERE

ASSOCIATIVA: $(a+b)+c = a+(b+c)$

e $5+(+\infty)=+\infty$

$$\underbrace{5+(+\infty)}_{\downarrow}-(+\infty)=5$$

? $0=(+\infty)-(+\infty)=5$?

LA SOMMA NON E' PIU'

ASSOCIATIVA

$$(a+b)+c \neq a+(b+c)$$

$$0 = (5+(+\infty))+(-\infty) \neq 5+(+\infty+(-\infty))$$

$$= 5$$

$$\frac{+\infty}{+\infty} = 1$$

$$5 \cdot \frac{+\infty}{+\infty} = 5$$

$$\underbrace{5 \cdot (+\infty)}_{+\infty} \cdot \frac{1}{+\infty} = 5$$

? $$1 = \frac{+\infty}{+\infty} = 5$$

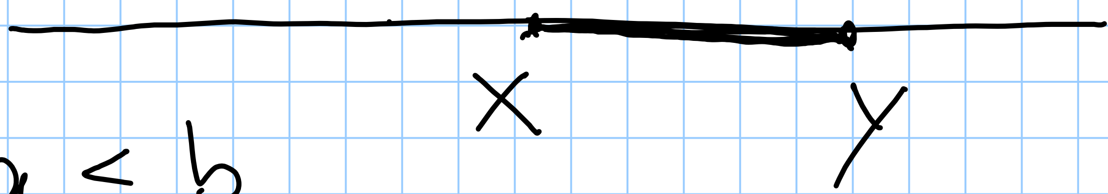
$$(a \cdot b) \cdot c \neq a \cdot (b \cdot c)$$

DEFINIZIONE

un INTERVALLO

$I \subseteq \mathbb{R}$ è un insieme:

$$x \in I,\ y \in I,\quad x < z < y \implies z \in I$$



$a \le b$

$$I = \{x \in \mathbb{R} : a \le x \le b\} = [a, b],$$ INTERVALLO **CHIUSO**

$$I = \{x \in \mathbb{R} : a < x < b\} = (a, b),$$ INTERVALLO **APERTO**

$$I = \{x \in \mathbb{R} : a \leq x < b\} = [a, b)$$

$$I = \{x \in \mathbb{R} : a < x \leq b\} = (a, b]$$

$\boxed{a = \inf I}$, $\boxed{\sup I = b}$

$a = -\infty$ $\quad (-\infty, b]$

$b = +\infty$ $\quad (a, +\infty)$

a e b SONO GLI **ESTREMI** di I

$$-\infty \leq a < b \leq +\infty$$

AMPIEZZA di $I := b - a$

$$A = \{x \in \mathbb{Q} : -1 \le x \le 1\}$$

$$= \mathbb{Q} \cap [-1, 1],$$ NON È UN INTERVALLO

$$A = \{x \in \mathbb{R} : -1 \le x \le 1\} =$$

$$[-1, 1]$$

$\mathbb{N}$ NON È UN INTERVALLO

X e Y INSIEMI NON VUOTI

INSIEME PRODOTTO

$X \times Y$ è L'INSIEME

DELLE COPPIE ORDINATE

degli ELEMENTI di $\mathbb{X}$ e $\mathbb{Y}$

$$\mathbb{X} \times \mathbb{Y} = \{(x,y) : x \in \mathbb{X}, y \in \mathbb{Y}\}$$

$$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 =$$

$$= \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}\}$$

$$\mathbb{R}^2 \ni (x,y) \neq (y,x)$$

$$\begin{array}{c} (1, 2) \neq (2,1) \\ \cap \quad \cap \\ \mathbb{R} = \mathbb{X} \quad \mathbb{Y} = \mathbb{R} \end{array}$$

DEFINIZIONE

SIANO X e Y due INSIEMI

UNA FUNZIONE da X in Y è UNA APPLICAZIONE CHE AD OGNI ELEMENTO di X ASSOCIA UNO ED UN SOLO ELEMENTO di Y.

L'INSIEME X SI DICE DOMINIO della FUNZIONE

L'INSIEME Y SI DICE CODOMINIO della FUNZIONE

$$f : X \longrightarrow Y$$

$$y = f(x) \quad , \quad x \in X$$

$y \in Y$

L'INSIEME degli ELEMENTI o VALORI in Y ASSUNTI da f è L'IMMAGINE di f

$$\mathrm{Im}(f) = \{ y \in Y : \exists x \in X : y = f(x) \}$$

$$\mathrm{Im}(f) \subseteq Y$$

SE $y \in \mathrm{Im}(f)$ e $y = f(x)$ $x \in X$, ALLORA x si DICE CONTROIMMAGINE di y (TRAMITE f)

L'INSIEME delle COPPIE ORDINATE $(x, f(x))$, $x \in X$ È IL GRAFICO di f.

$$\mathrm{Graf}(f) = \{(x,y) \in X \times Y : (x,y) = (x, f(x)),\ x \in X\}$$

$$\mathrm{Graf}(f) \subseteq X \times Y$$

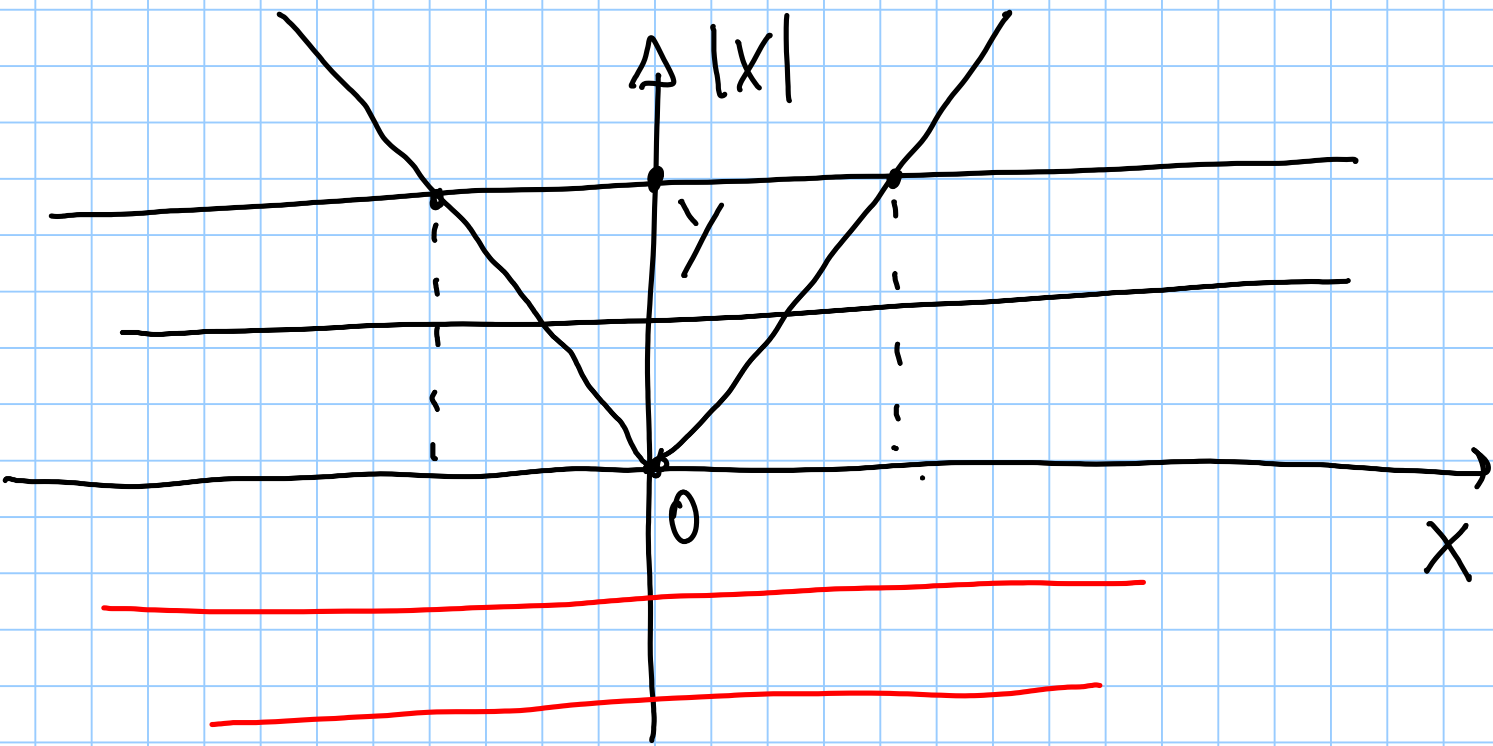
$$X \subseteq \mathbb{R}, \quad Y = \mathbb{R}$$

IL DOMINIO NATURALE di una funzione è L'INSIEME PIÙ GRANDE di NUMERI REALI IN CUI f è DEFINITA

$$\mathrm{dom}(f)$$

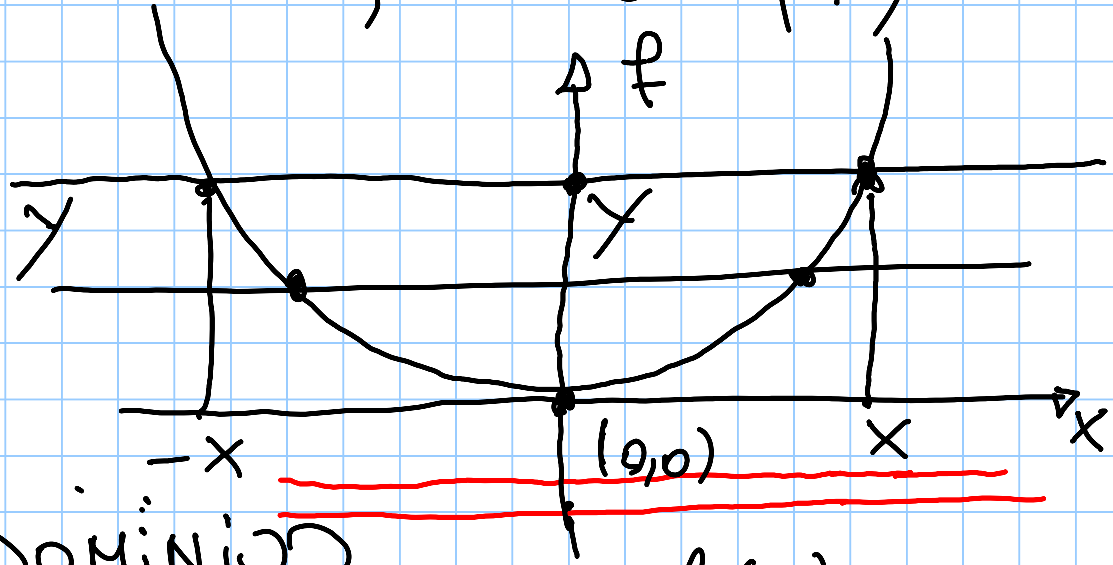
$f(x) = |x|$, $\mathrm{dom}(f) = \mathbb{R}$

$\mathrm{Im}(f) = [0, +\infty)$



Il codominio è $\mathbb{R}$.

$f(x) = x^2$, $\mathrm{dom}(f) = \mathbb{R}$



$\boxed{y \in \mathbb{R}}$

CODOMINIO

$f: \mathbb{R} \to \mathbb{R}$

$y = f(x)$

$f(0) = 0$

$[0, +\infty) \subseteq \mathrm{Im}(f)$

$y < 0$

$\Downarrow$

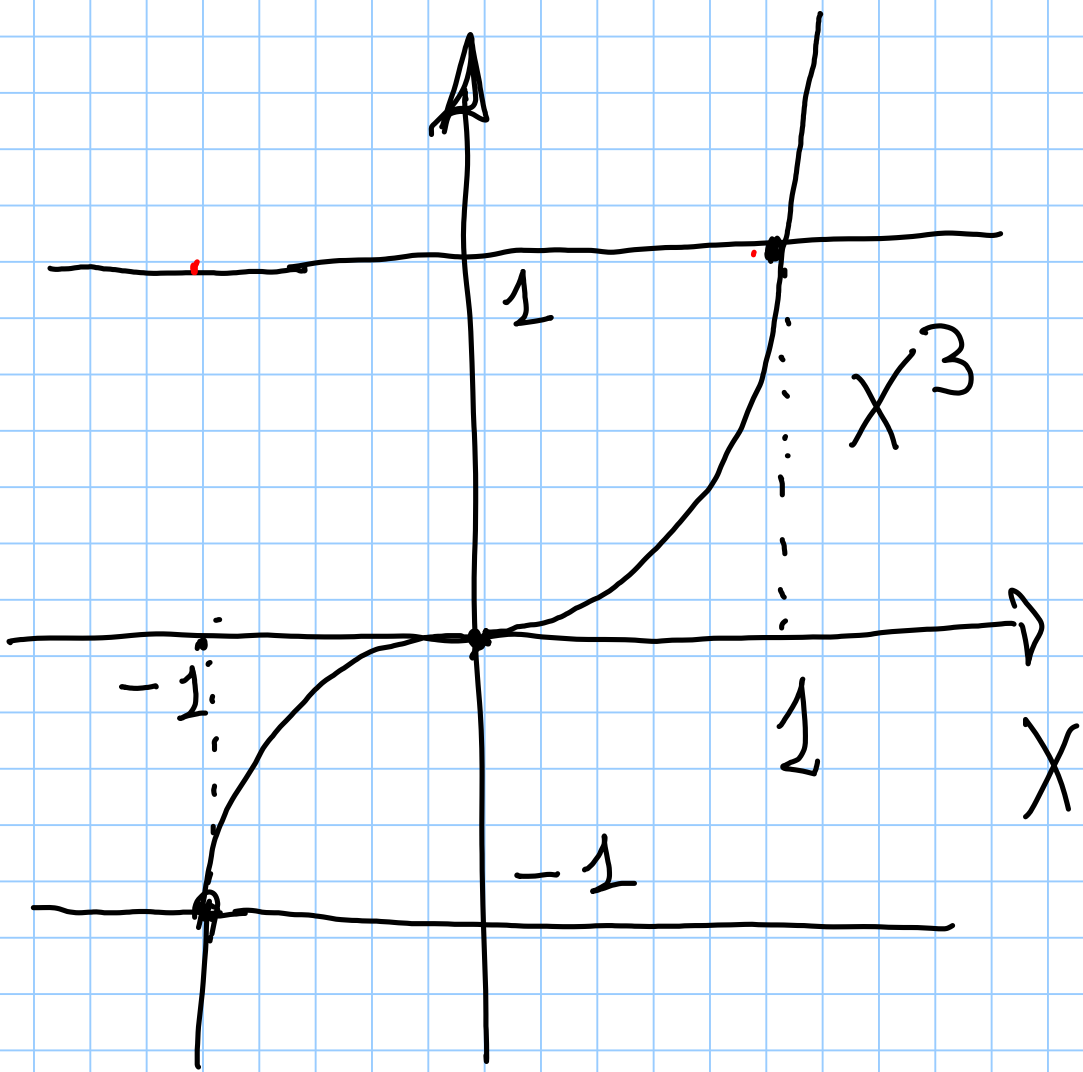
$y \notin \mathrm{Im}(f)$

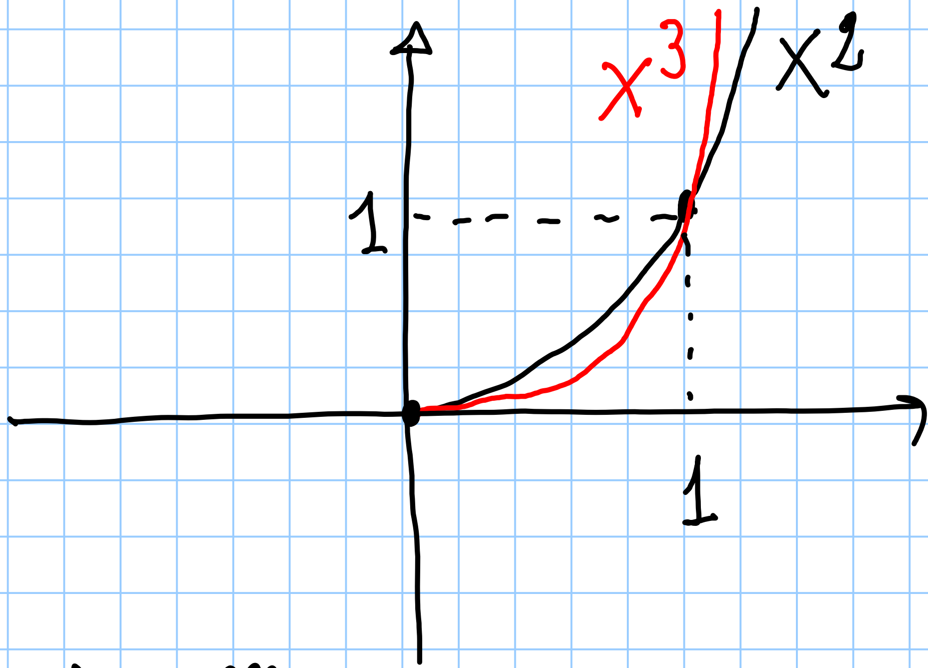
$\Downarrow$

$[0, +\infty) = \mathrm{Im}(f) \Longleftarrow y \notin \mathrm{Im}(f)$

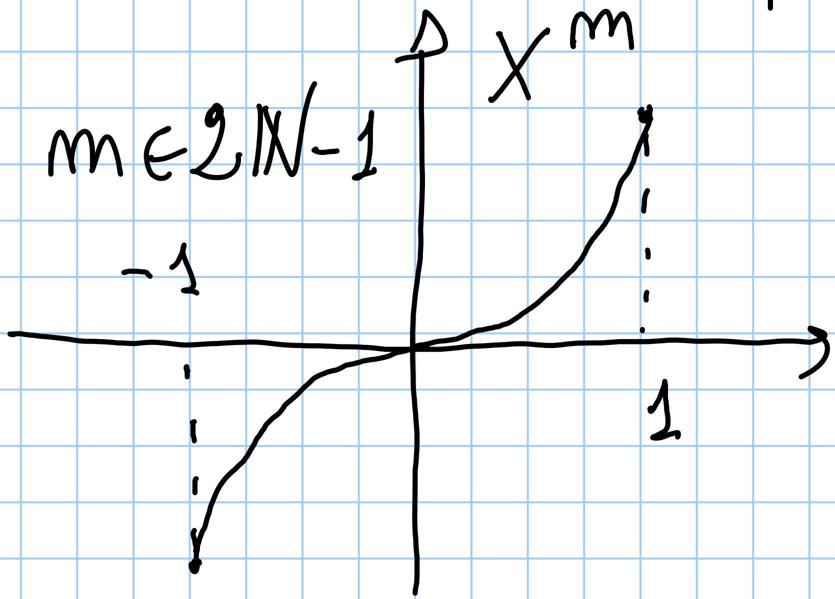
$f(x) = x^3$, $dom(f) = \mathbb{R}$

$Im(f) = \mathbb{R}$

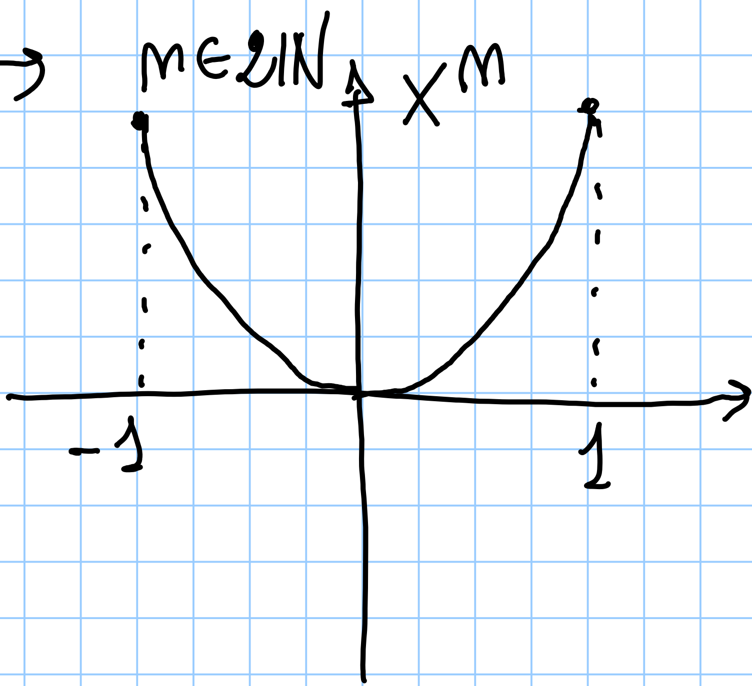




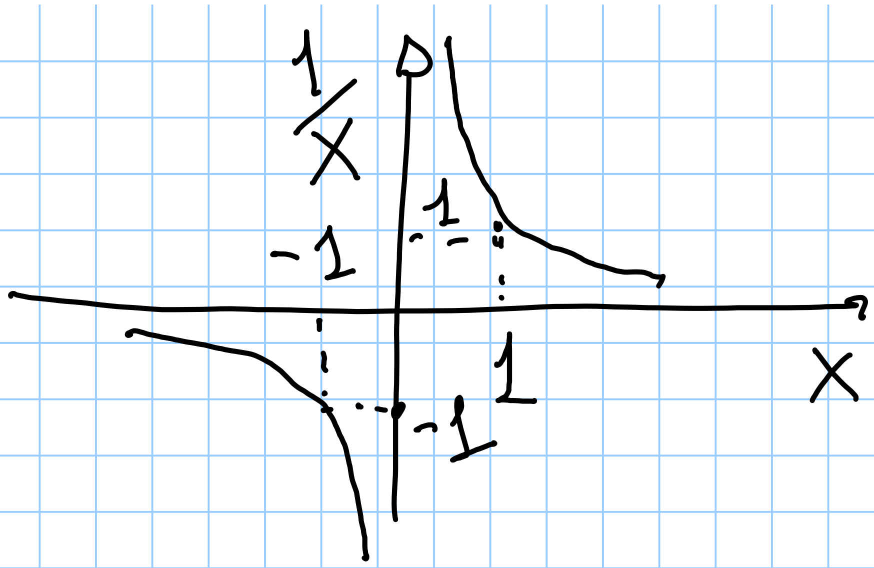
x^3
x^2
1
1



x^m
$m \in 2\mathbb{N}-1$
-1
1



$m \in 2\mathbb{N}$
x^m
-1
1

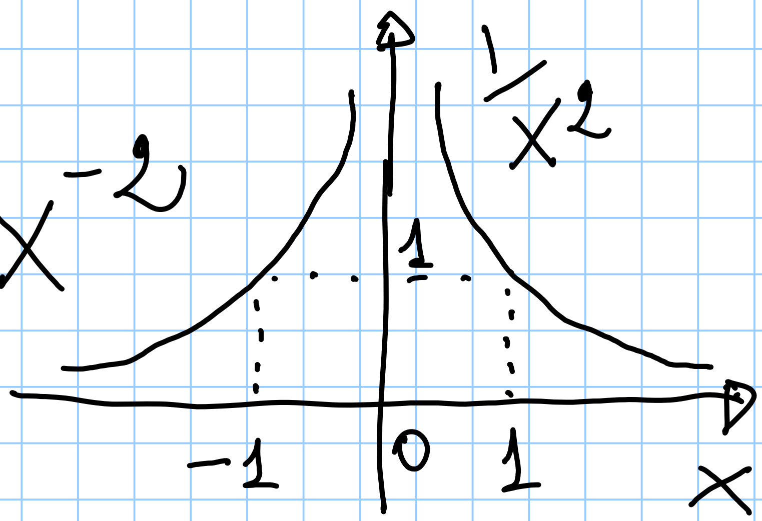


$$f(x) = \frac{1}{x} = x^{-1}, \quad dom(f) = \mathbb{R} \setminus \{0\}$$

$$dom(f) = (-\infty, 0) \cup (0, +\infty)$$

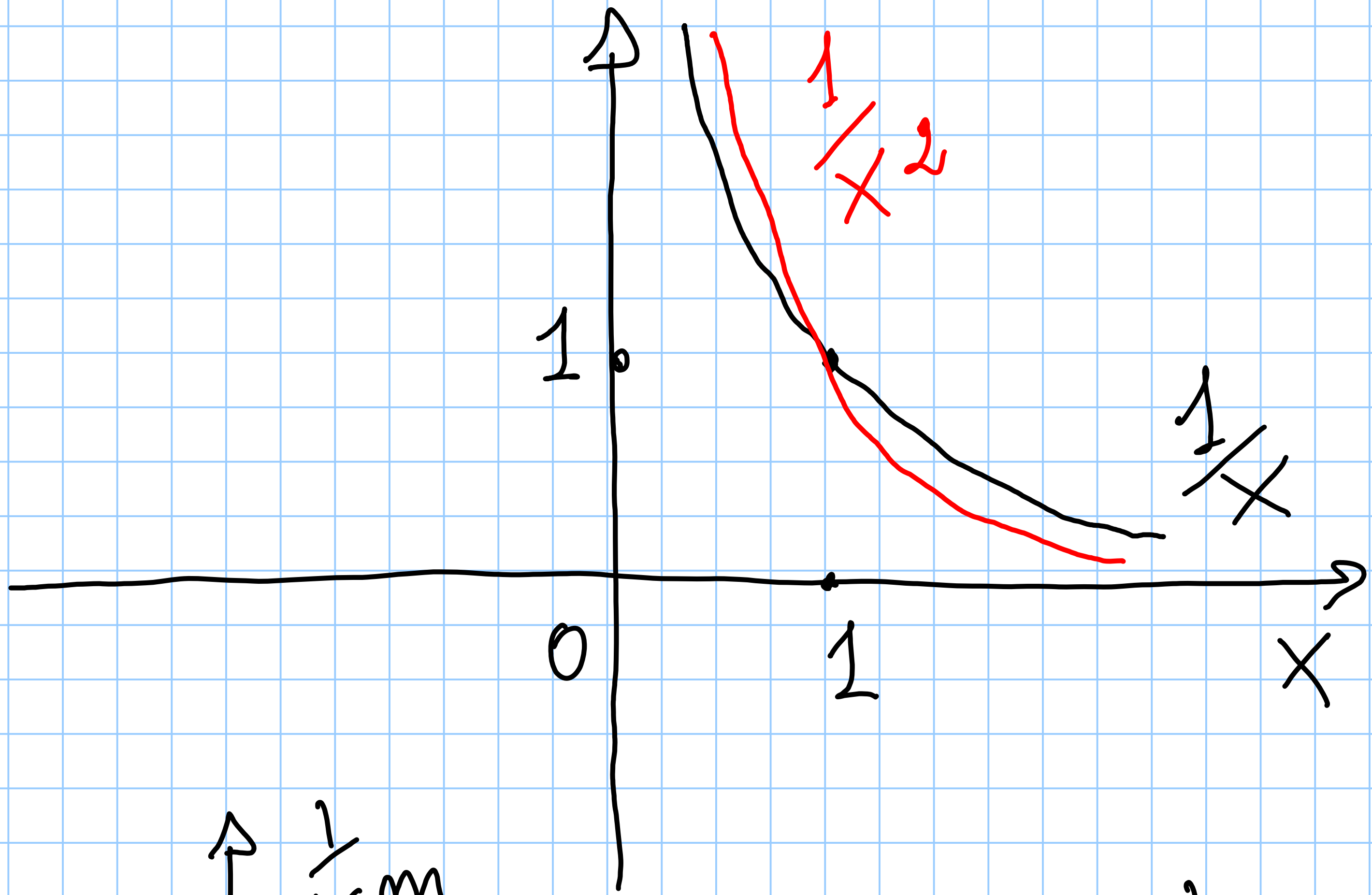
$$Im(f) = \mathbb{R} \setminus \{0\}$$

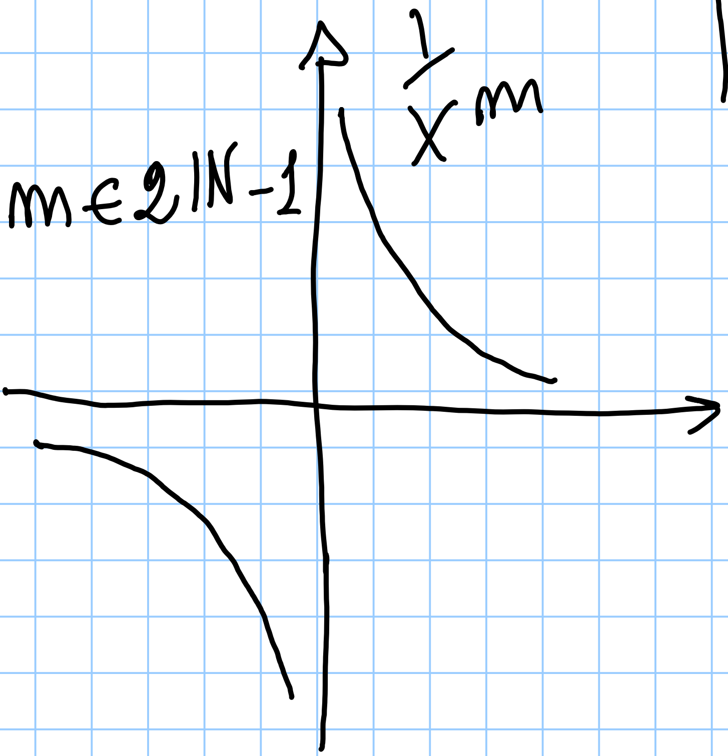
$$f(x) = \frac{1}{x^2} = x^{-2}$$

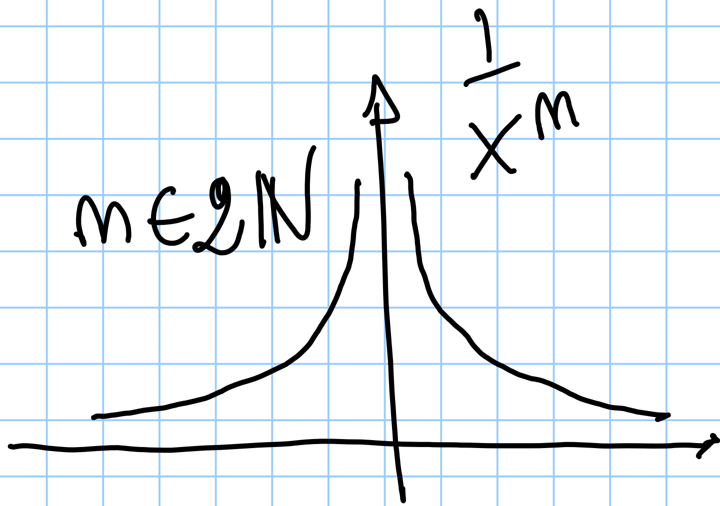


$$\mathrm{dom}(f) = \mathbb{R} \setminus \{0\}$$

$$\mathrm{Im}(f) = (0, +\infty)$$

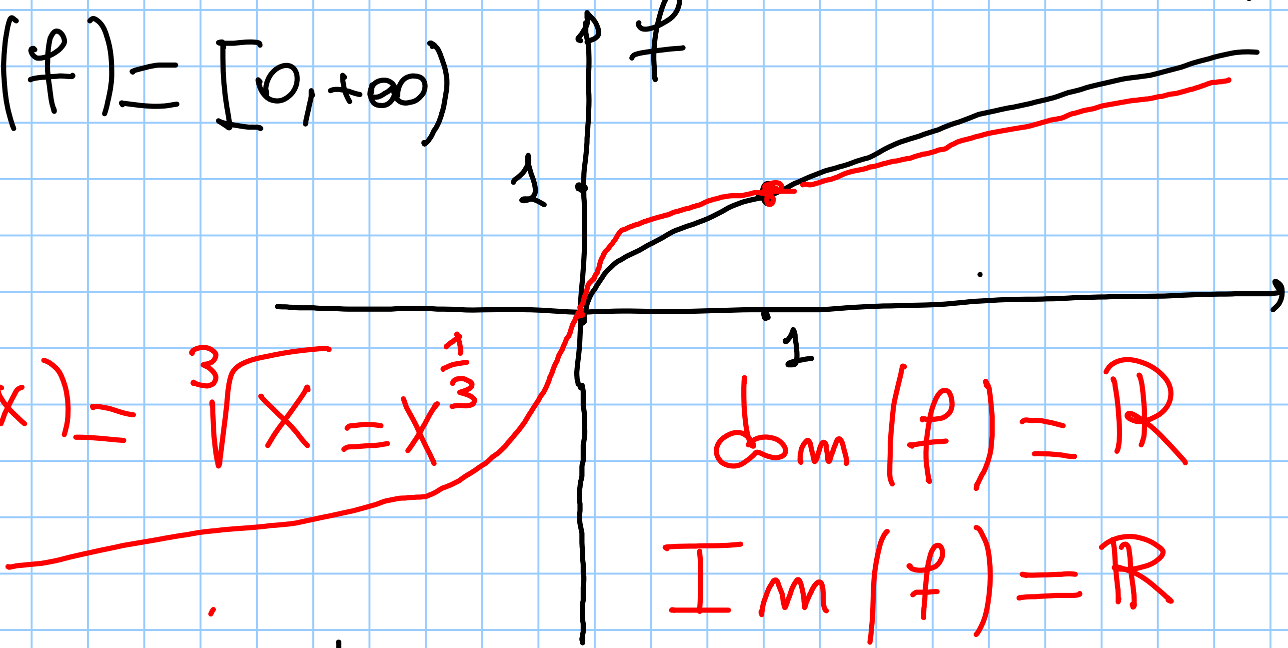

$1/x^2$
$1/x$
1
0
1
X


$m \in 2\mathbb{N}-1$
$1/x^m$


$m \in 2\mathbb{N}$
$1/x^m$

$f(x) = \sqrt{x} = x^{1/2}$, $\mathrm{dom}(f) = [0, +\infty)$

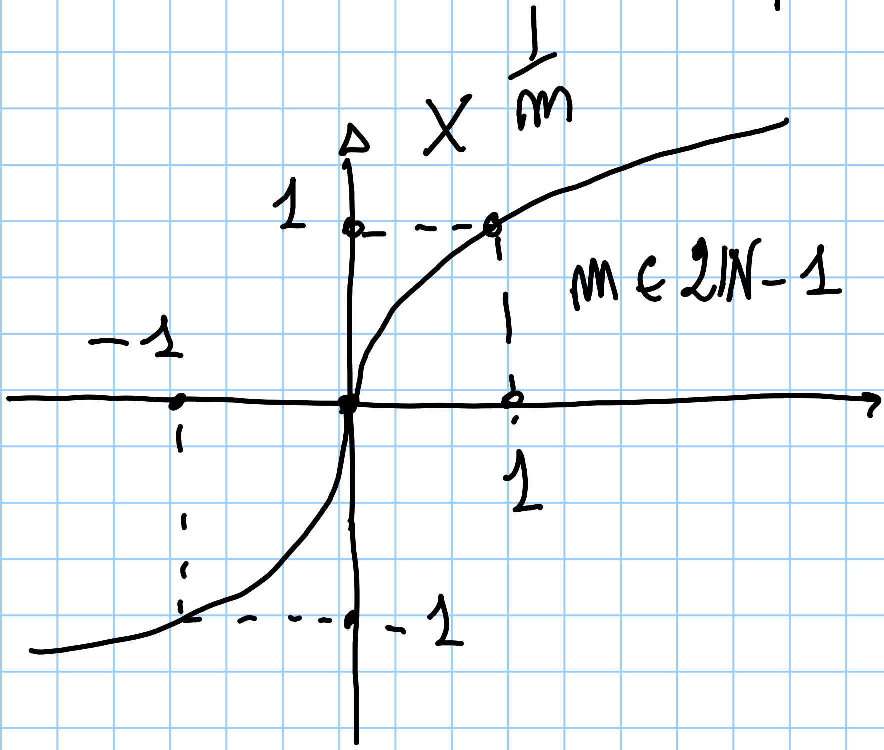
$\mathrm{Im}(f) = [0, +\infty)$

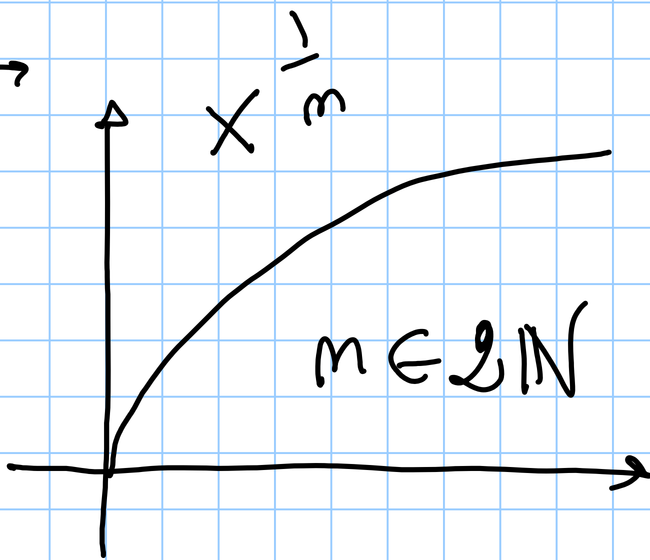


$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$\mathrm{dom}(f) = \mathbb{R}$

$\mathrm{Im}(f) = \mathbb{R}$

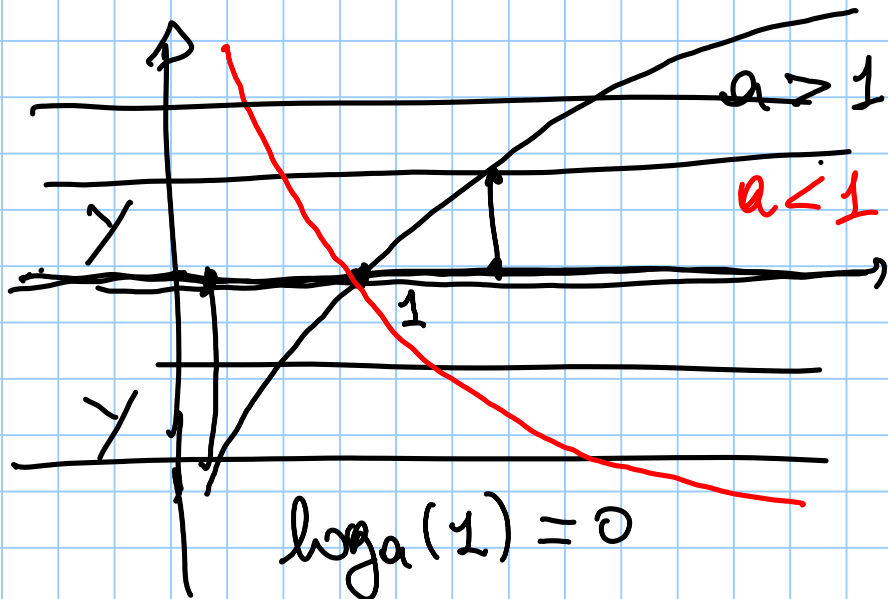




$f(x) = \log_a(x)$, $\mathrm{dom}(f) = (0, +\infty)$

$a > 0$, $a \neq 1$

$\mathrm{Im}(f) = \mathbb{R}$



$\log_a(1) = 0$

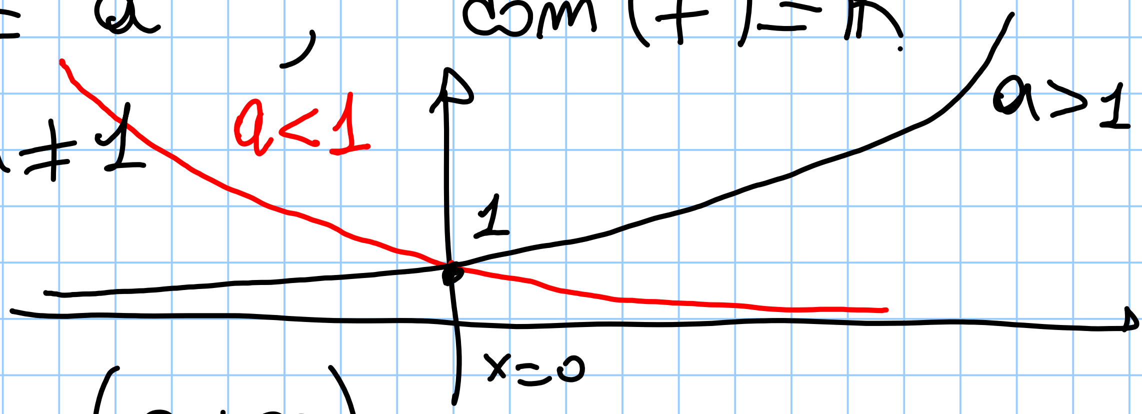
$y = \log_a(x) \Longleftrightarrow$

$a^y = x$ | $a^{\log_a(x)} = x$

$f(x) = a^x$, $\quad dom(f) = \mathbb{R}$

$a > 0$, $a \neq 1$

$a^0 = 1$

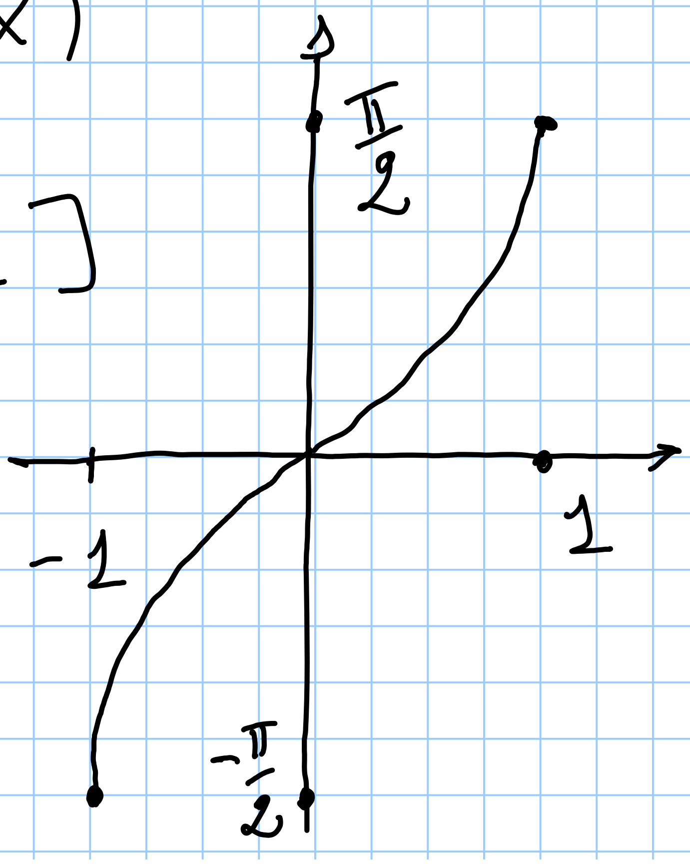


$Im(f) = (0, +\infty)$

$f(x) = \arcsin(x)$

$dom(f) = [-1, 1]$

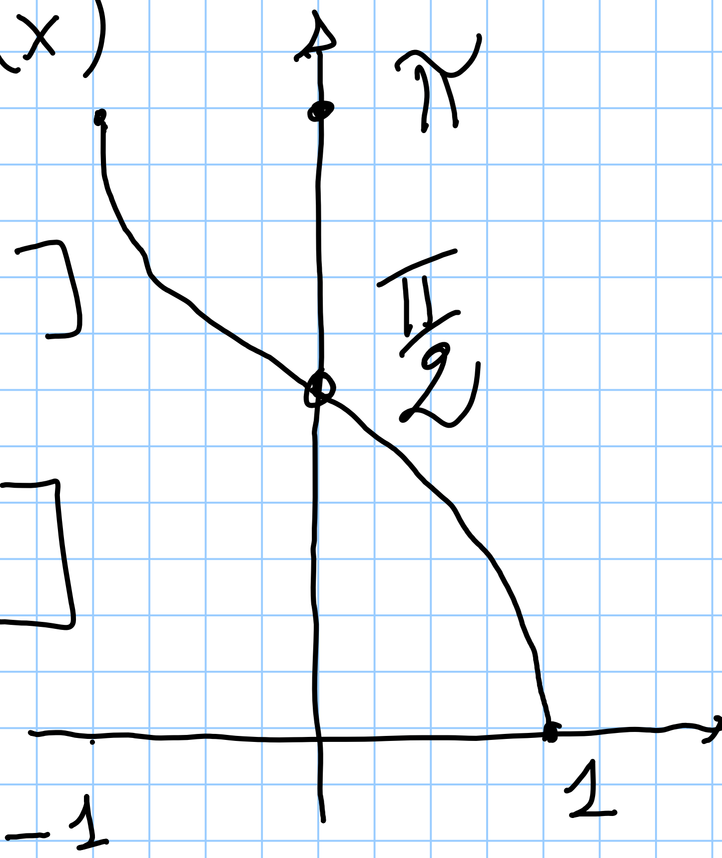
$Im(f) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$$f(x) = \arccos(x)$$

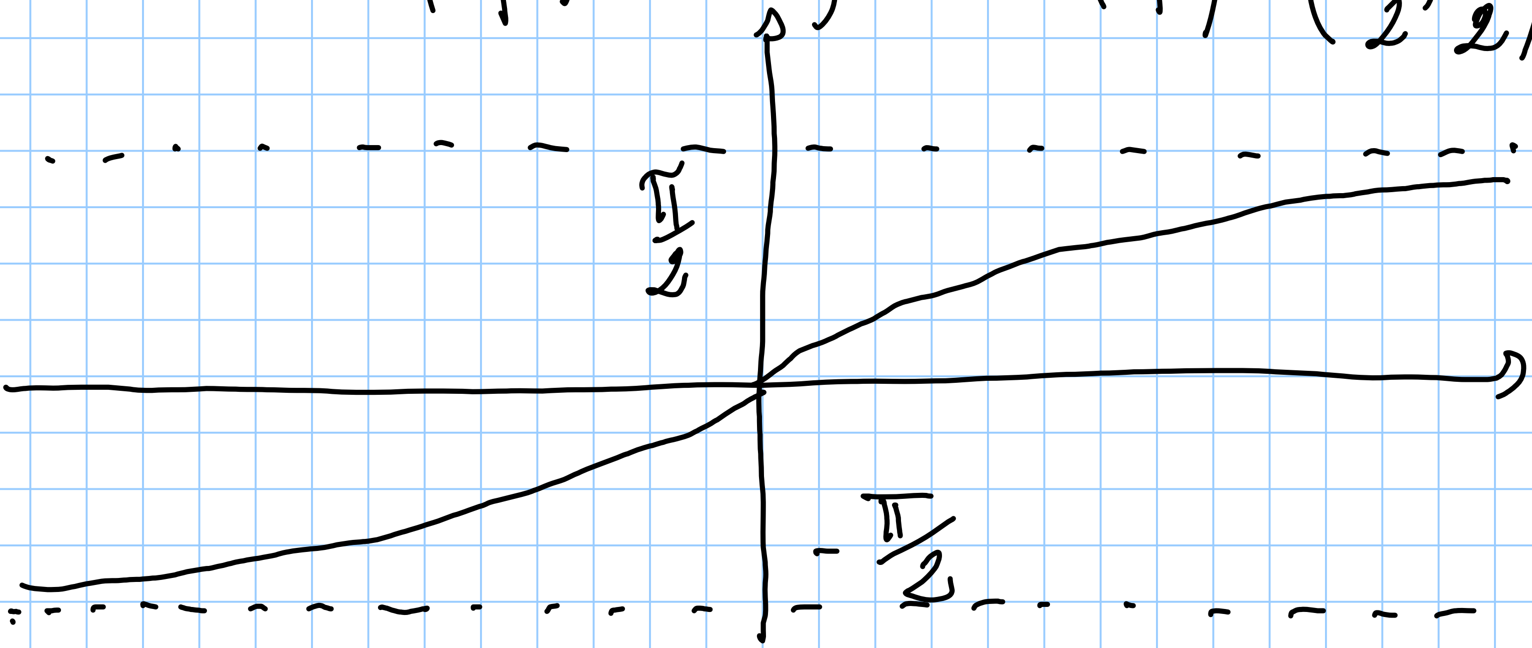
$$\mathrm{dom}(f) = [-1, 1]$$

$$\mathrm{Im}(f) = [0, \pi]$$



$$f(x) = \operatorname{arcTg}(x)$$

$dom(f) = \mathbb{R}$, $Im(f) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



$\boxed{\alpha > 0}$

$f(x) = x^{\alpha}$ $\quad dom(f) = [0, +\infty)$

$f(x) = x^{\frac{m}{n}}$ $\quad dom(f) = [0, +\infty)$

$\frac{m}{n} > 0$

$f(x) = x^{\frac{m}{n}}$, m e n primi tra loro

n dispari, $\frac{m}{n} = \frac{5}{7}$, $\frac{m}{n} = \frac{1}{3}$.

$$\mathrm{dom}(f) = \mathbb{R}$$

$$f(x) = x^{\frac{1}{3}} \quad , x \in \mathbb{R}$$

$$f(x) = x^{\frac{1}{5}} \quad , x \in \mathbb{R}.$$

APPENDICE

$$f(x) = x^{3/7} \Longrightarrow \mathrm{dom}(f) = \mathbb{R}$$

$$x^{3/7} = \left(x^{1/7}\right)^3$$

Sia $g(x) = x^{1/7}$.

DATO CHE (vedi sotto ⊛)

$\mathrm{dom}(g) = \mathbb{R}$, allora

anche $(g(x))^3$ è definita $\forall x \in \mathbb{R}$.

DATO CHE $f(x) = (g(x))^3$, allora $\mathrm{dom}(f) = \mathbb{R}$.

✻ Da quanto visto sopra, la PARTE DIFFICILE è VERIFICARE CHE $\mathrm{dom}(g) = \mathbb{R}$

Se $x \geq 0$ $g(x) = x^{\frac{1}{7}}$ è BEN DEFINITA $\Rightarrow$ $[0, +\infty) \subseteq \mathrm{dom}(g)$

Se $x < 0$, si ha $x = -|x|$ e

poniamo $y = |x|^{\frac{1}{7}}$.

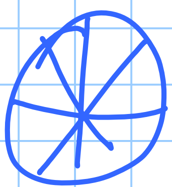
Si ha $(-y)^7 = \left(-\left(|x|^{\frac{1}{7}}\right)\right)^7 =$

$$(-1)^7 \cdot |x| = -|x| = x$$

Quindi se $y = x^{\frac{1}{7}}$, $x > 0$

allora $-y = (-x)^{\frac{1}{7}}$ e

in particolare $(-\infty, 0) \subseteq \mathrm{dom}(g)$.



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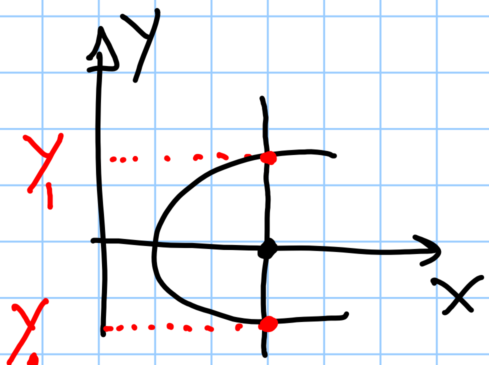

questo NON è il GRAFICO di una funzione.

$x \longrightarrow \{y_1, y_2\}$